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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

On the longitude of the sun in the years 1864—1900, and the rotation of the earth, by *W. de Sitter*.

In *B. A. N.* **124** it was found that from about 1865 to 1895 the longitudes of the sun taken from Dr. JONES's paper in *M. N.* lxxxvii, p. 4 show an apparent systematic error of about + 0″·6. In a conversation which I had lately at Greenwich with Sir FRANK DYSON and Mr. CULLEN, it was remarked that this interval practically coincides with that during which LEVERRIER's tables were used in the *Nautical Almanac*. Dr. JONES derived formulas for the correction: Leverrier *minus* Newcomb, which are given in *M. N.* lxxxvi, p. 428. Now experience shows that such corrections are always more or less subject to doubt *), and it would have been safer to compute the place of the sun for every month, as Dr. JONES did for the years 1836—1863. This is, of course, a rather considerable piece of work, and I have contented myself with taking a few samples distributed equally over the interval in question as well as over the different months of the year. The quantities $\Delta (L-N)$ given below are the differences: (*Nautical Almanac*—NEWCOMB's tables) *minus* (JONES's formulas *M. N.* lxxxvi, p. 428).

$\Delta (L-N)$

1864 Jan. 1	—0″·7	1876 Apr. 15	—0″·1	1889 May 15	—0″·4
65 Feb. 14	—0·7	78 Mar. 15	—0·1	90 Mar. 15	—0·3
67 May 15	—0·2	79 July 15	—0·3	92 Jan. 15	—1·1
69 Jan. 15	0·0	80 Dec. 15	—0·5	94 Oct. 15	—0·5
70 Aug. 15	—0·5	82 Oct. 15	—0·4	95 Aug. 15	—0·2
71 Nov. 15	—0·4	85 Apr. 15	—0·4	96 Sep. 15	—0·2
73 Feb. 14	—0·1	86 Sep. 15	—0·3	97 Mar. 15	—0·2
74 Jan. 15	—0·3	87 Dec. 15	—0·4	1901 Jan. 1	—0·3

The corrections are rather irregular, and do not show any decided correlation with the time. The general mean is

$$\Delta (L-N) = -0''{\cdot}36$$

If this is applied to the mean epochs 1869 to 1896·5 of *B. A. N.* **124**, TABLE 2, and one third of it to the two epochs 1863 and 1900·5. and if then a further correction of $-0''{\cdot}16\ T$ is applied to the mean longitude of the sun, the residuals become:

RESIDUALS OF SUN'S LONGITUDE.

1767	—1″·36	1863	—0″·12	1903·5	—0″·14
89	+ ·07	69	+ ·27	06·5	— ·10
1804	—1·52	73	+ ·16	09·5	+ ·14
17	—3·83	78	+ ·17	12·5	— ·04
27	—1·39	83	+ ·10	15·5	— ·16
34	+ ·19	88	+ ·13	18·0	— ·08
39	— ·09	92·5	— ·09	20·0	— ·16
47·5	— ·05	96·5	— ·34	22·0	— ·20
55·5	— ·05	1900·5	— ·21	24·0	— ·15
				26·0	— ·14

The residuals have become much less systematic. The residual for 1896·5, which was very small, has been made large negative, and that of 1900·5 also in a smaller degree. *) The correction applied is, of course, not entitled to great confidence, but the result seems to show that a more careful reduction would probably lead to still smaller residuals, and that consequently the interpretation of the observed fluctuations as given in *B. A. N.* **124** is confirmed.

It was shown in *B. A. N.* **124** that the observed fluctuations in the longitudes of the moon, the sun and the planets Mercury and Venus, can be explained by assuming two kinds of changes in the rotation of the earth, viz: (A) discontinuous changes of the moment of inertia, and (B) discontinuous changes of the coefficient of tidal friction. The cause (A) affects only the length of the day, the cause (B) also produces a change in the true mean motion of the moon, the consequent ratio of the changes in the apparent longitudes of the sun and planets and of the moon being $Q = Q_s$, while for the cause (A) it is $Q = 1$. According to *B. A. N.* **124**, p. 36, footnote, the most probable value of Q_s is $Q_s = 4{\cdot}4$.

Expressing the effect of both causes (A) and (B) in time, we adopt for the correction to be applied

*) *Cf.* NEWCOMB, *Astronomical Constants*, p. 7: "I can not but feel that the application of such corrections involves more or less doubt and uncertainty".

*) These two residuals depend on JONES's Table III (*M. N.* lxxxvii, p. 9), the others on his table II (p. 8).

to astronomical time in order to get uniform, or 'Newtonian' time:

Epoch of discontinuity	$\frac{\delta C}{C}$	Δt *by cause* (A)
		s
1663	$+ 1{\cdot}54.10^{-8}$	$- 27{\cdot}8$
1758	$- 0{\cdot}52$	$- 0{\cdot}5 + 48{\cdot}7\ (T + 1{\cdot}80)$
1784	$- 2{\cdot}06$	$+ 21{\cdot}4 + 32{\cdot}1\ (T + 1{\cdot}30)$
1864	$- 3{\cdot}09$	$+ 13{\cdot}9 - 33{\cdot}0\ (T + 0{\cdot}75)$
1876	$+ 1{\cdot}98$	$- 6{\cdot}5 - 130{\cdot}1\ (T + 0{\cdot}30)$
1897	$+ 3{\cdot}92$	$- 20{\cdot}4 - 67{\cdot}6\ (T + 0{\cdot}15)$
1918	$- 3{\cdot}92$	$- 23{\cdot}7 + 55{\cdot}8\ (T - 0{\cdot}05)$
		$- 17{\cdot}1 - 67{\cdot}6\ (T - 0{\cdot}20)$

For the cause (B) we have:

Epoch of discontinuity	Δt *by cause* (B)		$\frac{dE}{dt}$ *ergs per second*
	s	s	
1745	$+ 18{\cdot}8\ (T + 2{\cdot}073)$	$+ 43{\cdot}7\ S$	$- 3{\cdot}4.10^{19}$
1870	$- 14{\cdot}4\ (T + 1{\cdot}182)$	$- 23{\cdot}3\ S$	$- 1{\cdot}9$
	$- 46{\cdot}9\ (T + 0{\cdot}017)$	$+ 69{\cdot}2\ S$	$- 4{\cdot}7$
Average $= (B)_0$		$+ 40{\cdot}2\ S$	$- 3{\cdot}1$

The first straight line of (A) has been slightly altered, and the last added, and a smaller coefficient of S in the third parabola of (B) has been adopted. Otherwise these expressions are the same as those given in *B. A. N.* **124.** It will be remembered that $S = T^2 + 1{\cdot}33\ T - 0{\cdot}26$.

The effect of the cause (B) can be split up in the average $(B)_0$, which gives the secular accelerations, and the fluctuations $(B)' = (B) - (B)_0$. The excess of the longitudes of the sun and the planets Mercury and Venus over those given by NEWCOMB's tables are then:

$$\Delta L' = + 1''{\cdot}89 + 1''{\cdot}25\ T + 1''{\cdot}65\ S + 0{\cdot}0411\ [(A) + (B)']$$
$$\Delta\lambda_1 = + 7{\cdot}65 + 7{\cdot}13\ T + 6{\cdot}86\ S + 0{\cdot}171\ [(A) + (B)']$$
$$\Delta\lambda_2 = + 3{\cdot}30 + 2{\cdot}32\ T + 2{\cdot}70\ S + 0{\cdot}067\ [(A) + (B)'],$$

and for the moon the excess over BROWN's tables is

$$\Delta L = + 6''{\cdot}70 + 4''{\cdot}00\ T - 10''{\cdot}71 \sin (140^\circ{\cdot}0\ T + 240^\circ{\cdot}7) + 5''{\cdot}03\ S + 0{\cdot}549\ [(A) + 0{\cdot}23\ (B)'].$$

For the transits of Mercury the corrections ΔV and ΔW are as adopted in *B. A. N.* **124.**

The residuals given by these expressions are given in the following table.

The residuals for the sun and planets are in some cases even smaller than those found before, using the observed fluctuations of the moon instead of the computed (A) + (B').

RESIDUALS.

Moon Occultations				Moon Meridian obs.				Sun		Venus		Transits of Mercury			
												November		May	
t	Res.	t	Res.	t	Res.	t	Res.	t	Res.	t	Res.	t	Res.	t	Res.
								1767	$- 1''{\cdot}31$						
								89	$- 0{\cdot}02$						
1637	$+ 0''{\cdot}7$	1847·5	$+ 0''{\cdot}4$	1753·6	$+ 0''{\cdot}01$	1862·5	$+ 0''{\cdot}53$	1804	$- 1{\cdot}58$	1840	$- 1''{\cdot}14$	1677·9	$- 0''{\cdot}20$		
49	$+ 0{\cdot}2$	50·0	$- 0{\cdot}1$	59·2	$- {\cdot}72$	67·5	$+ {\cdot}19$	17	$- 3{\cdot}87$	45	$- {\cdot}60$	90·9	$- {\cdot}44$		
62	$+ 0{\cdot}8$	53·0	$+ 0{\cdot}28$	64·9	$- {\cdot}65$	72·5	$- {\cdot}24$	27	$- 1{\cdot}48$	50	$- {\cdot}30$	97·9	$- 4{\cdot}44$	1740·4	$- 13''{\cdot}39$
81	$- 0{\cdot}4$	57·0	$+ {\cdot}41$	70·6	$+ {\cdot}25$	77·5	$- {\cdot}47$	34	$- 0{\cdot}03$	55	$+ {\cdot}10$	1723·9	$+ 1{\cdot}14$	53·4	$+ 0{\cdot}06$
1710	$+ 0{\cdot}1$	61·0	$+ {\cdot}06$	76·2	$+ {\cdot}14$	82·5	$+ {\cdot}09$	39	$- {\cdot}33$	60	$+ {\cdot}24$	36·9	$+ 1{\cdot}14$	86·4	$- {\cdot}46$
27	$+ 0{\cdot}4$	65·0	$- {\cdot}61$	81·9	$+ {\cdot}52$	87·5	$- {\cdot}34$	47·5	$- {\cdot}18$	65	$+ {\cdot}18$	43·9	$+ {\cdot}17$	99·4	$+ {\cdot}12$
37	$+ 0{\cdot}4$	69·0	$+ {\cdot}44$	87·6	$+ {\cdot}30$	91·5	$+ {\cdot}01$	55·5	$- {\cdot}07$	70	$+ {\cdot}05$	56·9	$- {\cdot}05$	1832·4	$+ {\cdot}26$
47	$+ 0{\cdot}6$	73·0	$- {\cdot}65$	93·2	$+ 1{\cdot}92$	94·5	$- {\cdot}16$	63	$- {\cdot}14$	75	$+ {\cdot}35$	69·9	$- 1{\cdot}28$	45·4	$+ {\cdot}30$
55	$- 0{\cdot}1$	77·0	$- {\cdot}27$	98·3	$- {\cdot}07$	97·5	$+ {\cdot}16$	69	$+ {\cdot}19$	80	$+ {\cdot}47$	82·9	$+ 8{\cdot}24$	78·4	$+ {\cdot}48$
71	$+ 1{\cdot}0$	81·0	$- {\cdot}26$	1803·4	$- {\cdot}24$	1900·5	$- {\cdot}14$	73	$+ {\cdot}05$	84	$- {\cdot}11$	89·9	$- {\cdot}56$	91·4	$+ {\cdot}06$
85	0·0	85·0	$- {\cdot}19$	09·1	$+ {\cdot}14$	03·5	$+ {\cdot}03$	78	$+ {\cdot}03$	88	$- {\cdot}13$	1802·9	$- {\cdot}70$	1924·4	$- {\cdot}61$
92	$- 0{\cdot}3$	89·0	$+ {\cdot}48$	14·8	$- 1{\cdot}47$	06·5	$+ {\cdot}33$	83	·00	92	$- {\cdot}15$	22·9	$+ {\cdot}15$		
1801·5	0·0	93·0	$+ {\cdot}58$	20·4	$- 1{\cdot}69$	09·5	$- {\cdot}07$	88	$- {\cdot}02$	96	$- {\cdot}13$	48·9	$+ {\cdot}93$		
09·5	$+ 0{\cdot}9$	97·0	$+ 1{\cdot}01$	26·1	$- {\cdot}36$	12·5	$+ {\cdot}19$	92·5	$- {\cdot}19$	1900	$- {\cdot}15$	61·9	$+ 1{\cdot}31$		
13·0	$+ 0{\cdot}5$	1901·0	$+ {\cdot}04$	31·8	$- {\cdot}10$	15·5	$+ {\cdot}56$	96·5	$- {\cdot}42$	04	$- {\cdot}11$	68·9	$+ {\cdot}14$		
21·0	$+ 1{\cdot}0$	05·0	$+ {\cdot}46$	37·4	$- {\cdot}86$	18·5	$- {\cdot}30$	1900·5	$- {\cdot}31$	07	$+ {\cdot}07$	21·9	$+ {\cdot}40$		
22·5	$+ 1{\cdot}1$	09·0	$+ {\cdot}79$	43·1	$- 1{\cdot}01$	21·5	$+ {\cdot}83$	03·5	$- {\cdot}18$	10	$+ {\cdot}02$	94·9	$+ 1{\cdot}19$		
29·5	$- 0{\cdot}5$	13·0	$+ {\cdot}79$	48·8	$- {\cdot}31$	24·0	$+ {\cdot}10$	06·5	$- {\cdot}17$	13	$- {\cdot}06$	1907·9	$+ {\cdot}12$		
33·5	$- 0{\cdot}2$	17·0	$+ {\cdot}13$	52·5	$+ {\cdot}53$	26·0	$+ {\cdot}29$	09·5	$+ {\cdot}15$	16	$+ {\cdot}12$	14·9	$- {\cdot}06$		
39·5	$- 0{\cdot}6$	21·0	$+ {\cdot}35$	57·5	$+ {\cdot}75$			12·5	$+ {\cdot}01$	19	$+ {\cdot}19$				
								15·5	$- {\cdot}04$	22	$+ {\cdot}30$				
								18·0	·00						
								20·0	$- {\cdot}04$						
								22·0	$- {\cdot}05$						
								24·0	$- {\cdot}08$						
								26.0	$- {\cdot}07$						