# ON THE DETERMINATION OF EPHEMERIS TIME

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(Communicated by the Astronomer Royal)

(Received 1959 May 14)

*Summary*

It is pointed out that there are two inconsistencies in the set of values of $\Delta T$ given in the *Astronomical Ephemeris* for 1960: (i) a discontinuity at 1923 due to a change in the adopted equinox, and (ii) a change at 1923 in the value of the Earth's ellipticity used in computing the Moon's tabular place, which gives rise to an erroneous term with the period of the revolution of the node.

Recommendations are made for future discussion of lunar observations and the definitive determination of ephemeris time.

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1. *Introduction.*—The *Astronomical Ephemeris* for 1960 gives smoothed annual values of $\Delta T$ from 1900·5. Up to 1948·5 these values have been taken from a paper by Brouwer (1). The purpose of this note is to draw attention to certain inconsistencies in the reductions of the lunar observations which affect Brouwer's values of $\Delta T$, and to make some recommendations concerning the definitive determination of ephemeris time.

Before 1923 (when Brown's *Tables* were first used in the ephemerides) Brouwer's results depended systematically on the two occultation discussions of Spencer Jones (2), (3). The equations of condition for the two series were reduced by Spencer Jones to the system of Brown's theory by the application of the larger terms in the differences between the latter and Hansen's *Tables* on the one hand and Newcomb's "provisionally accepted theory" on the other. The Greenwich and Washington meridian observations for this period had also been reduced approximately to Brown's theory. However, Brouwer corrected these results empirically to the system of the occultations, so that any inconsistencies in the meridian reductions will have very little effect on his values of $\Delta T$.

Since 1923 the occultation results are those of the annual discussions, and, from 1932 onwards, the star positions used have been reduced to the system of the *Zodiacal Catalogue* (4), which is essentially the same as FK3. From 1923 to 1931 Brouwer corrected the occultations by an empirical linear formula obtained from comparison with the meridian observations.

2. *Ellipticity of the Earth.*—Although the occultations before 1923 have been reduced approximately to Brown's theory, the results derived from them differ in one important respect from those which would have been obtained if Brown's *Tables* had been used. The value of the ellipticity of the Earth used by Spencer Jones in both his discussions was 1/297, and the Greenwich and Washington meridian observations before 1923 were reduced with 1/297 and 1/298·2 respectively (5), (6). The value adopted by Brown in constructing his *Tables* was however 1/294. If it is assumed that 1/297 is correct, then, as has been pointed out elsewhere (7), adherence to 1/294 in the ephemeris will introduce an erroneous

term with a period of 18·6 years into $B$, the fluctuation in the Moon's mean longitude. It is thus desirable that Brouwer's values of $B$ after 1923 should be corrected by $+0''\cdot15 \sin \Omega$ where $\Omega$ is the longitude of the Moon's node, in order to bring them into accordance with those before that date.

In any future discussion of lunar observations, corrections should be applied to the ephemeris to reduce the tabular places to the best available value of the ellipticity. It would certainly be better to use the international value 1/297 rather than 1/294.

If $e$ is the true ellipticity, then an error in frequency measured in terms of the rate of change of $\Delta T$, obtained from comparison of observations of the Moon with the lunar ephemeris, is approximately $10^{-9}\,(e^{-1} = 294)\cos \Omega$. Recent observations of artificial satellites indicate that $e$ may be near 1/298 (**8**); the corresponding error in frequency will thus be $4 \times 10^{-9} \cos \Omega$.

3. *Equinox error.*—Brouwer states (**1**, p. 128) "... Newcomb's equinox... is the equinox used in Spencer Jones' revision." This statement appears to be erroneous. In the revision of Newcomb's occultations (**3**) a correction to the assumed right ascensions of the stars in Newcomb's fundamental catalogue was included in the equations of condition. We infer, therefore, that Spencer Jones' derived values of the mean longitude are independent of an error of equinox. The equinox correction which he derived for epoch 1850 was $-0^s\cdot047$; there is considerable uncertainty as to a possible mean motion, but we may take $-0^s\cdot05$ as the correction required by Newcomb's right ascensions at epoch 1900. The correction to the observed mean longitude corresponding to an equinox correction $E$ (seconds of time) is $15E \cos \epsilon$ where $\epsilon$ is the obliquity of the ecliptic. Thus for $E = -0^s\cdot05$ the correction to the observed mean longitude is $-0''\cdot69$.

The Cape occultations discussed by Spencer Jones (**2**) are referred specifically to Newcomb's right ascension system, and Brouwer found that an empirical correction of $-0''\cdot64$ was required to reduce them to the system of the revision of Newcomb's occultations. This is almost identical with the expected equinox correction. We may conclude, therefore, that before 1923 Brouwer's values of $B$ are all independent of an error of equinox.

Since 1923 the observations used by Brouwer are referred to adopted right ascensions which may be expected to be largely free from an error of equinox. However, Brouwer, supposing that his values of $B$ before 1923 were all referred to Newcomb's system, introduced a discontinuity of $+0''\cdot6$, which is equivalent to $+1^s\cdot09$, into $\Delta T$. All his values of $\Delta T$ since 1923 should thus be decreased by this amount.

4. *Definitive determination of* $\Delta T$.—The comprehensive discussion of Brouwer, as amended by the removal of the inconsistencies referred to above, contains the most homogeneous set of values of $\Delta T$ which are at present available.

Since 1923 the occultations have been reduced by lunations, and corrections obtained to the orbital longitude and latitude only. As has been remarked elsewhere (**7**), the long series from 1923 to 1959, extending over the whole period during which Brown's *Tables* have been in use, covers almost exactly two complete revolutions of the node. A discussion of these, and the meridian observations for the same period, should be made in order to obtain definitive values of $\Delta T$ as well as corrections to the orbital elements of the Moon. Such a discussion should include a possible equinox correction and mean motion. The importance of using the correct equinox, as distinct from one arbitrarily defined by a system of

adopted right ascensions, arises from the fact that an equinox correction $E$ results in a correction of approximately $25E$ to $\Delta T$, and a mean motion of this will affect the rate of change of $\Delta T$. It has been shown elsewhere (**7**) that the occultations give an equinox correction to FK3 which is in good agreement with that obtained from meridian observations of the Sun and planets.

Consideration should also be given to a rediscussion of all the lunar observations back to at least 1850 using an ephemeris rigorously computed from Brown's theory. Brouwer points out that the agreement between the meridian and occultation results has been very much better since 1923 than it was before that date, and attributes this partly to imperfect differential correction of the meridian observations from Hansen's *Tables* to Brown's theory. The marked increase in the scatter of the annual means from occultations before that date indicates that these too could be improved by such a rediscussion.

5. *Ephemeris Time.*—A practical system of uniform time is defined uniquely by two arbitrary constants, an epoch, and a unit of time. The epoch 1900 January $0^d\ 12^h$ Ephemeris Time (E.T.) is defined to be (**9**) "...the instant, near the beginning of the calendar year A.D. 1900 when the geometric mean longitude of the Sun was $279°\ 41'\ 48''{\cdot}04$...", and the unit of time is the ephemeris second as defined by the Comité International des Poids et Mesures (**10**), which is the time taken for the Sun's observed mean longitude to increase by $\frac{129\,602\,768''{\cdot}13}{3\,155\,760\,000}$ at epoch 1900 January $0^d\ 12^h$ E.T.; the numerator in this expression is the mean motion of the Sun in a Julian century as given in Newcomb's *Tables*, and the denominator is the number of seconds in a Julian century. The need for specification of the epoch in the definition of the unit of time arises because the actual motion of the Sun is accelerated.

The system of E.T. is thus defined uniquely in terms of two of the arbitrary constants of the Earth's orbit. The numerical values have been chosen to be the same as those adopted by Newcomb in constructing his *Tables*. If at some future date an ephemeris of the Sun with argument E.T. is to be constructed using an improved set of tables of the motion of the Earth, then the numerical values of these two arbitrary constants must be the same as those assigned by Newcomb.

Because E.T. is defined in terms of the mean longitude of the Earth, its practical determination must depend ultimately on observations of the Sun. But on account of the relatively small mean motion of the Sun, the I.A.U. has recommended the use of lunar observations and defined the quantity $\Delta T$ in terms of $B$ (**11**). The two time systems E.T., and U.T. $+\Delta T$ are however not logically the same. The determination of $\Delta T$ from the lunar ephemeris depends on the constants of the Moon's mean longitude used in that ephemeris, so that before $\Delta T$ can be used as an approximation to E.T.–U.T. these arbitrary constants must be determined from observations. A further complication in using lunar observations is the existence of an empirical secular retardation term in the expression of the Moon's tabular mean longitude (**12**); this has been emphasized by Atkinson (**13**).

In his fundamental work on the rotation of the Earth (**14**), Spencer Jones related the mean longitudes of the Sun, Mercury and Venus to that of the Moon. This work is based on observations extending from the latter half of the 17th century to 1936. For the greater part of this period, values of $B$ were taken from his discussion of Newcomb's occultations (**3**). However, from 1908 his values depend partly on Greenwich meridian observations and partly on the annual

occultation discussions, and from this date onwards there is a systematic difference of nearly 1″ between the values he used and Brouwer's values amended for the error of equinox referred to above. A new discussion of the modern observations of the Sun and planets using definitive values of $\Delta T$ from the rediscussion of the lunar observations would improve the determination of ephemeris time over the last hundred years. An important consequence of such a discussion would be a new determination of the empirical secular term in the Moon's mean longitude.

6. *Conclusions.*—The points raised in this note may be summarized in the following specific recommendations :—

(i) A comprehensive discussion of all lunar observations from 1923–1959 should be made in order to derive definitive values of $\Delta T$, as defined in (**11**), and corrections to the elements of the Moon's orbit.

(ii) All lunar observations from 1850 to 1922 should be rediscussed using an ephemeris of the Moon computed from Brown's theory.

(iii) Each of the above discussions should be based on the best available value of the Earth's ellipticity.

(iv) The derived values of the fluctuations in the Moon's mean longitude should be freed from errors of equinox.

(v) The Sun and planet observations from 1850 should be rediscussed using the values of $\Delta T$ obtained in (i) and (ii) above to give a definitive determination of ephemeris time, as defined in (**9**).

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1959 *May* 12.

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