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THE ACCURACY OF TUCKERMAN'S SOLAR AND PLANETARY TABLES

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The solar, lunar and planetary tables of Bryant Tuckerman[1] have now been available for more than fifteen years. During that time they have proved of inestimable value to historians of astronomy. In two remarkably compact volumes, these tables give the celestial coördinates of the Sun, Moon and five bright planets (Mercury, Venus, Mars, Jupiter and Saturn) at 5- or 10-day intervals over the entire period from 601 B.C. (= −600) to A.D. 1649.

The present paper grew out of a query put to one of us (F.R.S.) some years ago by an eminent historian of science regarding the real accuracy of the tables. For the Sun and planets, positions are given to the nearest 0°·01. Is this precision artificial or not? At the time it was impossible to answer this query satisfactorily since no purely gravitational ephemeris was available for all the planets to provide an independent check. The recent production of the Jet Propulsion Laboratory *Long Export Ephemeris* (DE 102) now makes such an undertaking viable. In our investigation, we have tested the solar and planetary data given by Tuckerman against the JPL ephemeris (hereafter abbreviated to *JPLE*) on some 200 randomly selected dates over the entire interval from −600 to +1649. This paper is devoted to summarizing and discussing our results with the user of Tuckerman's work particularly in mind.

Construction of the Tables

Rather surprisingly, Tuckerman's tables of the Sun, Moon and inner planets (including Mars) are based on what was already outmoded theory. He preferred to use the theory of Leverrier[2] for the Sun and inner planets and that of Hansen[3] for the Moon. For the two outer planets, Tuckerman used Gaillot's theory.[4] None of these theories has formed the basis of the *Astronomical ephemeris* and *American ephemeris* in the last fifty years or more. However, it is not our main purpose here to criticize Tuckerman's choice of theory. It is the tables themselves which are our major concern. These have justifiably become a standard work and have never been superseded.

Tuckerman chose geocentric ecliptical coördinates (celestial longitude and latitude). This was a sound choice, for equatorial coördinates (right ascension and declination) are more difficult to interpolate. Except in the case of the Moon, all positions are expressed to the nearest 0°·01. This accuracy is sufficient for almost all practical purposes except possibly for comparison with some of the early telescopic observations. To aid interpolation, coördinates are given at 5-day intervals for the faster moving planets Mercury and Venus and at 10-day intervals for the Sun and slower moving planets. Lunar positions are given at 5-day intervals, but to an accuracy of 0°·1. Higher frequency and greater precision, although desirable, would have been impractical on account of the rapid and irregular motion of our satellite. Nevertheless, the lunar tables are far less versatile than their solar and planetary counterparts; for example,

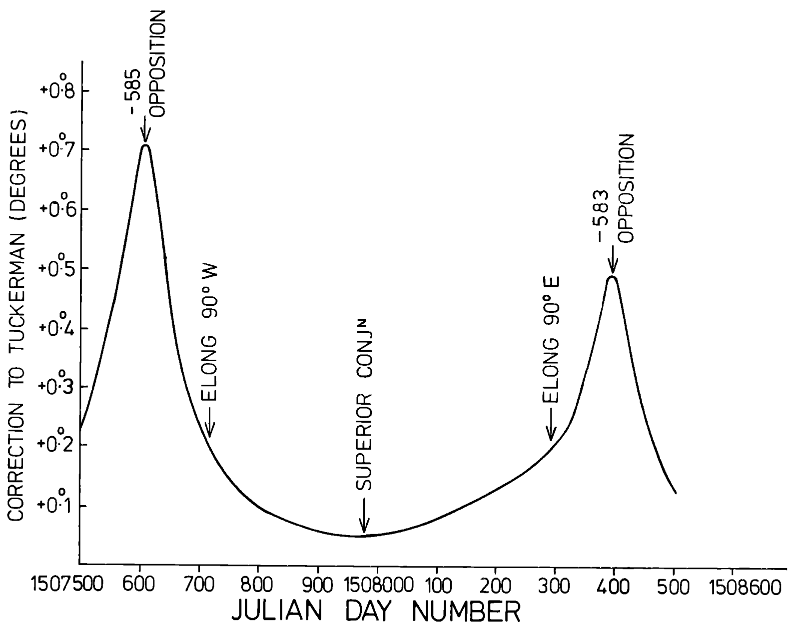

FIG. 1. Corrections to Tuckerman's tabular longitudes of Mars as deduced from the JPL *Long Export Ephemeris* (*JPLE*) over a specimen 1000-day interval (−585 to −583).

they cannot be used in accurate eclipse calculations. We have decided, therefore, to restrict our investigation to Tuckerman's data for the Sun and planets.

Throughout his work, Tuckerman used the Julian calendar. The correction to the Gregorian calendar for the period from +1582 to +1649 is thus 10 days. All *major* dates (at 10-day intervals) have Julian Day Numbers divisible by 10, a convenient choice. The time-system adopted was Universal Time (UT) which is most useful when dealing with historical data. UT is, of course, measured by the rotating Earth so that it is a very practical system. Its only drawback is the irregularity of the rotation of the Earth. The time of day selected by Tuckerman was 16^h UT, which corresponds to early evening (7 p.m.) Babylon/Baghdad time.

As regular users of Tuckerman's work, we can offer only one major criticism with regard to presentation. It would have been extremely valuable if planetary apparent magnitudes had also been available, especially for Mercury and Mars, whose brightness fluctuates dramatically. These magnitudes might have been readily computed using the empirical formulae of Müller,[5] which incidentally are still used in calculating the data for the *Astronomical ephemeris*.

The JPL Long Export Ephemeris

The *JPLE* is based on a numerical integration of the equations of motion of the planets using Cowell's method.[6] In essence, this consists of calculating the total force acting on each planet due to the gravitational attraction of the Sun and all the other planets, then calculating the next position and velocity by integrating the equations of motion. At the end of each step, new rectangular coördinates are obtained, which enables the gravitational forces to be recalculated and another step taken. Cowell's method is often preferred because it is easy to program on a computer, but great care is needed to obtain high accuracy.

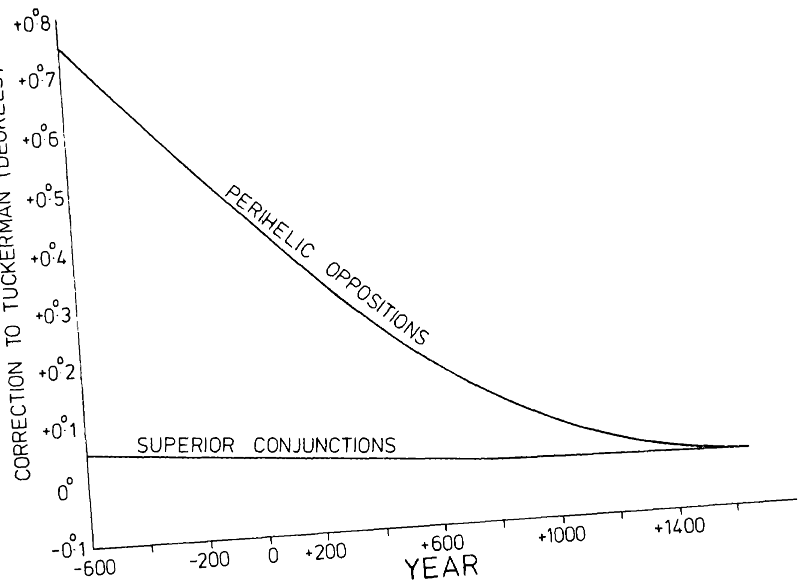

FIG. 2. Corrections to Tuckerman's tabular longitudes of Mars at perihelic oppositions (maximum discrepancy) and superior conjunctions (minimum discrepancy) in order to agree with longitudes computed using the *JPLE*.

The first problem, peculiar to Cowell's method, is that the solar term usually dominates in the total force calculation. Hence many figures have to be carried in the arithmetic to incorporate correctly the much smaller planetary terms. A second factor is that *all* numerical integration procedures suffer from truncation and rounding errors. Truncation error arises because one is solving a system of *difference* equations rather than the original differential equations. Rounding error on the other hand arises because of the unavoidable rounding errors in the arithmetic in each step. These cause the building up of small random effects which do not average out as the integration proceeds. A third problem is that there is no generally accepted way to define an optimum step size for the integration. Generally, the smaller the step size the more accurate the result, but the cost of computer time is a major factor. Additionally, there comes a point when too small a step size can increase the rounding error in the final result.

The principal advantage of an integrated ephemeris is that a 'theory' for planetary motion, such as used to construct Tuckerman's tables, is not needed. Given accurate starting positions, velocities and masses for the planets, then in principle an ephemeris can be calculated at any time in the future or past. This *will* contain errors at some level, the magnitude of which can only be found, in the final analysis, by direct comparison with observation.

In the modern epoch, the *JPLE* is believed to be accurate to 70 km or so, the order of 0·1 arcsec at 1 AU. However, we are not aware of any published comparison between it and ancient planetary observations. We have made a check on the *JPLE* results against an earlier published integration of the outer planets (Jupiter and Saturn) back to 1653.[7] Comparison of the computed rectangular coördinates for earlier dates indicates that the agreement is satis-

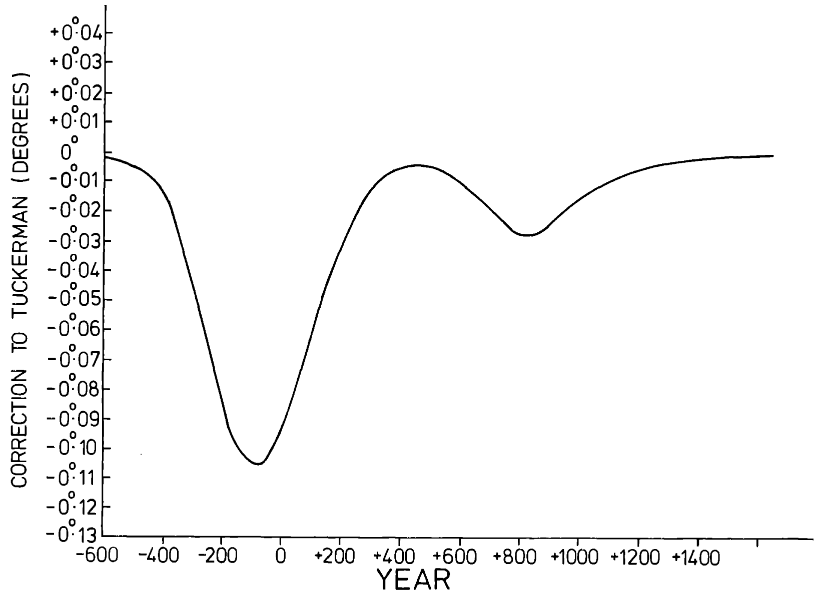

FIG. 3. Mean corrections to Tuckerman's tabular longitudes of Jupiter in order to agree with longitudes computed using the *JPLE*.

factory down to the fifth decimal place. This is also quite typical of the agreement at other more recent randomly selected dates. The corresponding maximum *angular* discrepancy is about 10 arcsec which is negligible for the present purpose.

The *JPLE* is supplied on six magnetic tapes, with a set of FORTRAN subroutines to extract the planetary positions and velocities (referred to epoch 1950·0) for any date from −1410 (JDN 1206160·5) to +3002 (2817872·5).

Analysis of Tuckerman's Data with the JPL Ephemeris

In order to make direct comparison with the positions given by Tuckerman, times were converted from UT to their equivalent Ephemeris Time (ET) by the following expression:

$\Delta T = +4^{s}{\cdot}87 + 35^{s}{\cdot}06\ T + 36^{s}{\cdot}79\ T^2$ (T in Julian centuries from 1900·0) which we derived by comparing Tuckerman's expression for the solar mean longitude with that of Newcomb, which defines ET.

As might be expected, the planetary *latitudes* are almost invariably accurate to the precision quoted; only Venus moves very far from the ecliptic (to a maximum of about 8°). However, the longitudes tabulated by Tuckerman require detailed discussion. For Mercury, Venus and the Sun, the agreement in longitude between Tuckerman and the *JPLE* is excellent. Discrepancies of more than 0°·02 as far back as −600 are very rare and further are essentially random. The maximum error discovered in our sample of 200 positions was 0°·05 (for Venus). On the contrary, the deviations for Mars are disturbingly large. The maximum error found was more than 0°·7, even more than the Moon's apparent diameter! Figure 1 illustrates a test on the longitude of Mars over a randomly chosen 1000-day interval (−585 to −583). This clearly shows that the discrepancy is correlated with the synodic period of Mars (780 days).

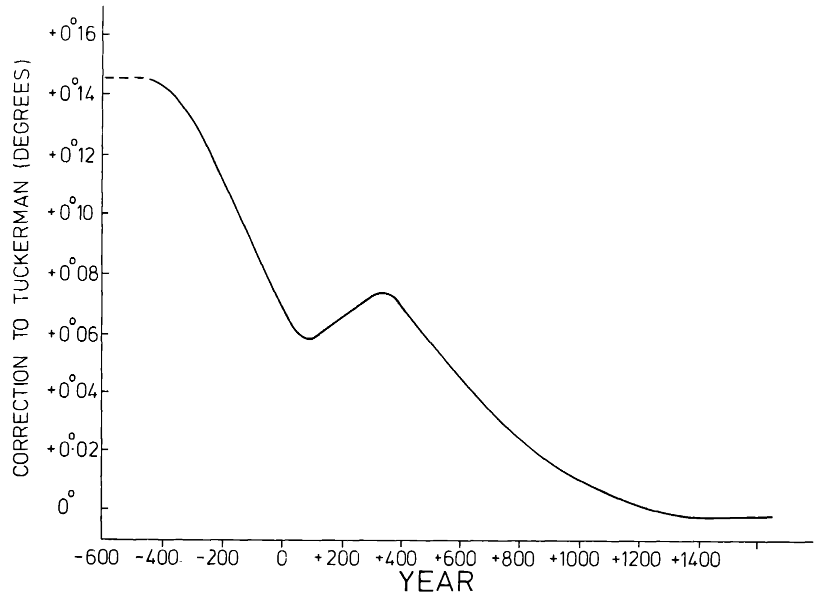

FIG. 4. Mean corrections to Tuckerman's tabular longitudes of Saturn in order to agree with longitudes computed using the *JPLE.*

Near superior conjunction the correction is tolerable—some 0°·05—but near opposition (where the planet is of course brightest), the deviation is intolerably high and rapidly changing. The magnitude of the deviation is clearly a function of the distance of the planet from the Earth: at the −585 opposition the planet was much closer to the Earth (0·385 AU) than at the −583 opposition (0·50 AU). The scale of these discrepancies is such that we are unable to offer a convenient correction table without recomputing the Mars longitudes for the whole interval of the tables.

Figure 2 shows the maximum and minimum discrepancies down the centuries. On any given date, the actual discrepancy could be anywhere between the two curves.

For the outer planets Jupiter and Saturn, the agreement between Tuckerman and the *JPLE* is very satisfactory. For Jupiter, the maximum discrepancy is about 0°·1 around the epoch −100 (see Figure 3). It is evident from Figure 3 that the corrections to accord with the *JPLE* are approximately periodic with increasing amplitude going backwards in time. The graph itself was obtained by drawing a smooth curve through about 200 scatter points. It should allow sound working corrections to be estimated—to an accuracy of better than 0°·02. In the case of Saturn, a fairly smooth correction graph can be constructed in a similar manner to that for Jupiter. Use of Figure 4 should allow estimates of the longitude of the planet to within about 0°·03 of the *JPLE* value except before about −300. For the first three centuries of the tables the scatter is rather large—approaching 0°·1.

Conclusion

For all the planets but Mars, Tuckerman's tables either give directly, or can be reduced in a simple form as outlined above to provide, substantial agreement with the JPL *Long Export Ephemeris*. (For the period −600 to −300, longitudes of Saturn show a significant, but still fairly acceptable deviation.) The situation for Mars is quite different. Before about A.D. 1000, *i.e.* most of the period of the tables, large and rapidly varying discrepancies prevail. These are frequently measurable with the unaided eye. We suggest that production of new tables for Mars is a matter of some urgency.

Acknowledgements

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6. For an illustration of this method see R. M. L. Baker, *Astrodynamics* (New York, 1967).
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NOTE ADDED IN PROOF

B. Emerson of the Royal Greenwich Observatory has kindly supplied a selection of early coördinates for Mars computed using a program based on the theory of Newcomb and Ross. This program was developed by Mannino *et al.* (G. Mannino, L. Dall'Olio and L. D'Ascanio, "Programma per il calcolo delle effemeridi dei Pianeti Interni", *Pubblicazioni dell'Osservatorio astronomico universitaro di Bologna*, ix (1965), n. 2). The agreement between these positions and the equivalent JPL data is of the order of 0°·01 as far back as −600. This result strengthens our confidence in the reliability of the *JPLE* figures.