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Secular variation of the Moon's rotation rate

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Abstract. The Moon is responsible for an appreciable tidal bulge on the Earth. This bulge has a lag angle with reference to the lunar direction, responsible for the secular acceleration of the Moon. This paper presents the resulting action on the lunar spin motion. First, the direct effects of the anelastic terrestrial tides on the Moon's rotational motion have been modeled and isolated. The resulting lunar libration is characterized by a long modulation with a period of 80 years. The advance of the terrestrial tidal bulge relative to the lunar direction modifies only slightly this modulation. Second, a spin-orbit interaction has been isolated from the spin-orbit coupling included in our global model of the lunar motion. This interaction, permitted by the lunar spin-orbit resonance 1/1 and related to the terrestrial bulge advance, is characterized by a secular slowing down of the Moon's rotation rate. The mechanism of the secular acceleration of the lunar longitude is matched by the secular acceleration of the lunar rotation, preserving the synchronous rotation ($\dot{n} \simeq \dot{\omega} \simeq -25''/cy^2$ and $\dot{\omega}/\omega \simeq -14.10^{-9}\, cy^{-1}$).

Key words: celestial mechanics – methods: numerical – Moon – Earth

1. Introduction

A very accurate accumulation of Earth-Moon distance measurements has been collected by the Lunar Laser Ranging experiment (*LLR*) (see for instance Newhall et al. 1983, Veillet et al. 1989). Current state of the art LLR data reach accuracies at the 2- to 3-centimeter level, averaged over the data sets of the last few years (Dickey et al. 1994). It should be however emphasized that a careful choice of the best performing nights during the lastest years attain even better accuracy, about 1 to 2 centimeters, and significant improvement to a level of a few millimeter is now in progress at the Grasse station (Samain 1995). In terms of lunar librations, the quality of this current best data means the level of a few milliarcseconds (*mas*) and better can be reached in the future. Such observations concern the Earth-Moon System Geophysics and Dynamics (see for instance Ferrari et al. 1980, Williams et al. 1973, 1978) and certain aspects of fundamental physics (see for instance Nordtvedt 1973 and 1988, or a review by Mulholland 1980). However, in order to analyse these measurements, to de-correlate the dynamical mechanisms they intrinsically contain, to understand and explain these mechanisms, to delimit the internal structure of the Moon etc., a theoretical model is needed consistent with the accuracy of the observations. Conversely, such accuracy is then useful to determine a constrained and well-ajusted theory capable of giving an effective and real meaning to its descriptions, refuting or validating its results.

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The analytic, semi-analytic and numerical methods of Celestial Mechanics each have their advantages and their difficulties often with different aims. We may find in Eckhardt (1981) a synthesis of this question for the case of the representation of the Moon's motion. These three ways of proceeding contribute to the general improvement of celestial mechanics and to a better physical understanding of the phenomena. Concerning the highly precise theoretical representation of lunar motions in agreement with the very accurate LLR data, it is necessary to carry out high-performance numerical integration processes (cf. for instance Newhall et al. 1983) in parallel to the physical comprehension established by analytical means. Many perturbations are difficult to include in analytical theories. For instance, the planetary perturbations do not appear directly as disturbing torques in the existing analytical and semi-analytical theories of lunar rotation but indirectly through the orbital motion (for example Eckhardt 1981 and 1982; Migus 1980; Moons 1982 and 1984). Such perturbations can easily be numerically integrated. The same situation occurs if spin-orbit couplings are required, notably related to mutual potentials and tides. Moreover, the secular acceleration of the lunar rotation being a spin-orbit interaction, as studied in this paper, only a simultaneous spin-orbit integration of the global differential system contains implicitly such a phenomenon. The analytical and semianalytical theories of lunar spin do not integrate directly spin-orbit couplings, even if, from a quantitative point of view, an accurate interpolation may be performed from an orbital ephemeris, or else if it is possible to add to the solution some approximate terms in t^2 for instance. On the other hand, such spin-orbit couplings are present in the JPL ephemeris based on a simultaneous numerical integration. However, in this paper, we wish to isolate specifically this lunar spin-orbit interaction related to terrestrial tides.

The study of tidal effects is of great importance in the dynamic evolution of solar system bodies. Their list is preponderant for understanding the different resonance states. In particular, the synchronous spin-orbit motion of the Moon presents various interactions between physics and dynamics related to tidal mechanisms. The Moon is indeed responsible for an appreciable tidal bulge on the Earth. Connected to anelastic effects, this bulge has a lag angle with reference to the lunar direction. The behaviour of the lunar librations caused by this terrestrial bulge has been determined and plotted over 24 years in a previous study (Bois & Journet 1993). Over this interval of integration, the amplitudes were negligible but the long period effects were not reached. In particular, the nutation angle θ seemed to have a secular variation. In the present work, this mechanism is firstly studied on a larger interval of time in order to complete the previous behaviour. Secondly, the simultaneous effect of the terrestrial bulge advance on the coupled spin-orbit motion of the Moon is taken into account and described. Consequences on the lunar rotation rate are presented.

The question of the lunar orbit's secular acceleration has been largely discussed since the eighteenth century. In return, the behaviour of the secular slowing down of the lunar rotation needed in our opinion, to be isolated from the spin-orbit coupling included in a global model of the Moon's motion.

2. Theory

An accurate theory of the Moon's spin-orbit motion has been constructed in accordance with the requirements of current observational accuracy. The approach consists in integrating the n - body problem on the basis of the gravitation description given by Einstein's general relativity theory. A global formulation of general relativity for celestial mechanics problems has been recently given in the work of Damour et al. (1991, 1992, 1993) dealing with the global dynamics of n arbitrarily composed and shaped, weakly self-gravitating, rotating, deformable bodies; the formalism being derived from the first post-Newtonian approximation level. Relativistic patterns stem from these works. Gravitational figures of the bodies are represented by complete expansions in spherical harmonics (Borderies 1978; Schutz 1981). Internal structures of solid bodies, homogeneous or with core-mantle interfaces, are represented by the parameters of elastic and anelastic deformation (Eckhardt 1981; Lambeck 1980; Lefftz et al. 1991).

The model, solved by modular numerical integration, is controlled in function of the different physical contributions and parameters taken into account. Computations have been performed in quadruple precision (32 significant figures, integration at 10^{-16} internal tolerance) in order to avoid numerical divergence at the level of our tests (confirmed by backward integrations). The programmed theory can evolve in the sense that it can easily incorporate future extensions. Let us notice that the three physics (fundamental physics, celestial mechanics, and continuum mechanics) required and used to perform the theory do not arise and behave with the same level of control and expert appraisement. The gravitation concept arises as a provided *product* stemming from a background theory. The celestial mechanics problem is written, in the present theory, under a complete and wholly algorithmic form. For instance, formally, any figure-figure and tidal interaction can be stimulated in the model. As for the continuum mechanics representation, the phenomena cannot be taken into account exhaustively. The global description squares more or less well with the very different local configurations (Earth and Moon, core-mantle interfaces, etc.) or with modeling limited by nature (for instance atmospheric pattern). In fact, the geophysical aspects of the Earth interaction with the Moon constitute the main problem regarding the precision and coherence of the present spin-orbit theory of the Moon with respect to the observations, notably relative to the significance of the relativistic contributions.

3. Method

Nevertheless, we have established a suitable modular and autonomous environment initiated by the parameters stemming from recent data reductions. The two modes of lunar motion, spin and orbital, the n body problem (for the n planet motions of the solar system, the Earth included), and the gravitational and tidal interactions are then simultaneously integrated with the choice of the contributions and truncations at our disposal. For instance, the upper limits of the extended figure expansions and mutual interactions may be chosen as follows: up to $l = 5$ in the Moon case, 4 for the Earth, 2 for the Sun while only the Earth-Moon quadrupole-octupole interaction is taken into account. The model has been especially built to favour a systematic analysis of all the effects and contributions. The method of analysis allows the identification of relationships between causes and effects including interactions between physics and dynamics. In particular it permits the separation of various families of lunar librations as shown in previous papers (Bois et al. 1992; Bois & Journet 1993; Bois & Vokrouhlický 1995).

The non-linear features of the differential equations, the degree of correlation of the studied effect with respect to its neighbours (in the Fourier space) and the spin-orbit resonance, in the lunar case, make it hardly possible to speak about 'pure' effects with their proper behaviour (even after fitting of the initial conditions). The effects are not absolutely de-correlated but relatively isolated. The technique used (modular and controlled numerical integration, differentiation method and frequency analysis) may give the right qualitative behaviour of an effect and a good quantification of this effect relative to its neighbours (see Sect. 4 and Table 1). It is also possible to make some comparisons with analogous problems (for instance between analogous torques acting on the Moon and Earth's rotations). When a rotational effect is simply periodic, a fit of the initial conditions for a set of given parameters only refines without changing completely the effect's behaviour.

The model uses at present the dynamical parameters of the JPL DE303 ephemeris, including the Moon's spherical harmonics up to the fourth degree. Those of the fifth degree are found in Ferrari et al. (1980). These two adopted sources of parameters are mutually coherent. The first four significant figures

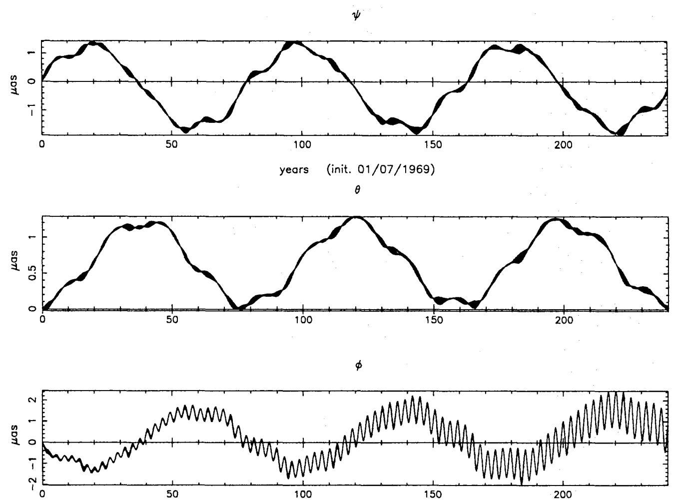

Fig. (1a–c). Terrestrial tidal librations of the Moon as they formally appear in the series of the Euler angles characterizing the lunar rotation. The differences ($\Delta\psi, \Delta\theta, \Delta\phi$) are obtained with respect to the solution free of the direct torque due the terrestrial tidal bulge acting on the Moon. Microarcseconds are on the vertical axis and years on the horizontal axis. Dominant terms of 80.1 year periodicity are in all Euler angles which contain also a slight distortion of 18.6 years. The ordinary resonant frequency of 2.9 years for physical librations in longitude is clearly visible in the ϕ angle of lunar proper rotation.

of common parameters are identical, which is largely sufficient for the studies presented below. Different numerical tests and experiments have shown that the behaviour of the curves and results obtained in the present study is globally insensitive to a refinement of the values of the parameters. Let us also note, that the initial date of our integrations coincides with that of the JPL DE303 ephemeris. In order to justify the consistency of our theory, we have adjusted it to the JPL ephemeris on the first 1.5 years up to a level of a few centimeter residuals ($\simeq 3cm$), simply by fitting the positions and velocities of the Earth and Moon. Besides, the computation of the lunar orbit's secular acceleration presented below (in Fig. 3 Sect. 6), gives us a good internal test in relation to consistency of the theory.

4. Lunar physical librations

In the theory, three Eulerian angles (ψ, θ, ϕ) stemming from the classical 3-1-3 angular sequence serve to represent the local rotational motion of the Moon with respect to a lunar reference system referred to a terrestrial equatorial frame (J2000), according to the following decomposition and notation: the precession ψ (around a fixed axis OZ, from a reference axis OX), the nutation θ (around an intermediate axis pointing toward the ascending node and representing the inclination of the Oz body-fixed axis with respect to the OZ-axis), and the rotation ϕ (around Oz).

The lunar physical librations have been studied on several occasions such as in Eckhardt 1981, Migus 1980, or Moons 1982. Let us recall the major changes in complete libration angles during the considered time span: the lunar nutation angle θ changes between 22 and 25 degrees with the major periodicity of 18.6 years driven by the nodal period of the lunar orbit; the proper rotation of the Moon around its axis of figure is affected by the complete angle ϕ with the mean period of 27.3 days; the precession angle ψ is also locked in resonance with the lunar node period of 18.6 years. Further forced behaviour due to the indirect action of the planets and higher harmonics of the Moon can be found in Eckhardt (1982) or Moons (1984).

A first aim was to include in the model all phenomena up to the precision level resulting from the LLR data, and if possible even better for reasons of consistency (i.e. at least 1 *cm* for the distance, 1 *mas* for the librations). In particular, several phenomena capable of producing effects of at least 0.1 *mas* in the lunar physical librations have been modeled and analyzed (the resulting libration may be at the observational accuracy level). Other libration effects smaller than this threshold of accuracy have been nevertheless included and studied because of their qualitative interest.

The nature, cause and behaviour of the Moon's physical librations have been then isolated and described by using the model and the method summarized in the previous section. The direct effects resulting from planetary actions (essentially Venus) and Earth-Moon figure-figure interactions have been presented in Bois et al. (1992). Physical librations due to lunar and terrestrial tides have been expanded and described in Bois & Journet (1993). Relativistic contributions related to the rotational motion of the Moon, as well as the relativistic precession of the Earth reference frame have been presented in Bois & Vokrouhlický (1994, 1995). In order to get an idea about the importance of these different contributions, Table 1 presents a comparison of their amplitude maxima reached in each case by one of the three Eulerian angles ($\Delta\psi, \Delta\theta, \Delta\phi$). Results are given for three fundamental periods of librations (when they have been estimated), i.e. the two resonant frequencies of the spin-orbit motion of the Moon (Kopal 1969; Yoder 1981), namely 2.9 and 80.1 years, and the period of 18.6 years forced by the nodal motion of the Moon. When the 2.9 year period is excited, the effects occur exclusively on the libration in longitude represented by the ϕ angle. The acronym *ExMy* signifies orders of multipoles respectively in interaction, namely the order *x* of the Earth acting on the order *y* of the Moon. *LTL* means lunar tidal librations and *RCL* refers to the relativistic contribution librations. Let us note for instance that the amplitude of the relativistic contributions is comparable with the quasi-Newtonian interaction of the Earth quadrupole and the Moon octupole (E2M3) or else a little less than the effects of the lunar tides.

Because of the growing analysis of many kinds of imbricate librations or perturbations with some ambiguous historical points, in particular in the Moon and Earth cases, a discussion on librations has been carried out in a previous work (Bois 1995), leading to establish a general classification of rotational swing motions by way of a new terminology proposal. The classification is compact. Two great libration families serve to define the physical librations. For libration sub-classes, the designation method is extended to any identified mechanisms. Besides, another definition of the ambiguous designation of "free libra-

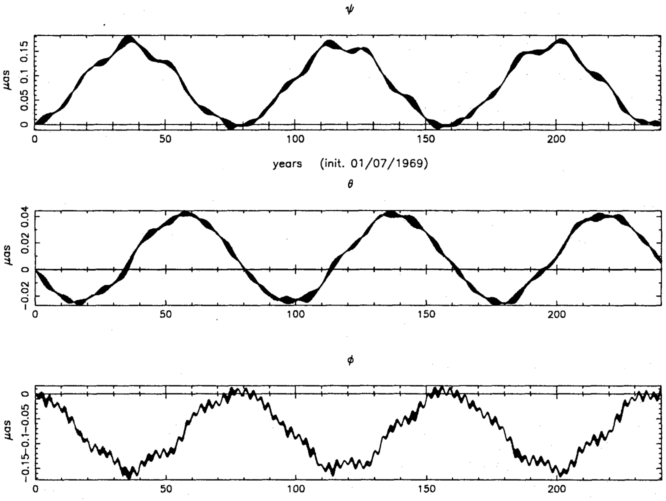

Fig. (2a–c). Signature of the advance of the terrestrial tidal bulge on the Moon's rotational motion represented by the classical Eulerian angles (3-1-3 angular sequence). The differences $(\Delta\psi, \Delta\theta, \Delta\phi)$ are obtained by considering the direct torque due the terrestrial tidal bulge acting on the Moon but, with respect to Fig. 1, keeping k_2 fixed and varying ϵ between the two integrations. Microarcseconds are on the vertical axis and years on the horizontal axis. The global behaviour is the same as in Fig. 1, with the same periods, but with amplitudes at least one order of magnitude lower (and a phase difference in time).

Table 1. Comparison of various effects on the Moon's librations (See the text for terminology).

effect	2.9 years	18.6 years	80.1 years	units
V0M2		≤ 3		*mas*
E0M5	10	≤ 20		*mas*
E2M2	≤ 10	45	45	*mas*
E2M3	0.5	≤ 1		*mas*
LTL	1	3		*mas*
RCL	0.1	1	1	*mas*

tion" is proposed. The so-called free librations and the historical debate (still remaining open) on their excitation sources require particular attention (see for instance Yoder 1981 and Eckhardt 1993). A forthcoming paper will focus on two mechanisms, internal to the Earth-Moon system, adequate to excite the two resonant frequencies of 2.9 and 80.1 years : the spin-orbit coupling and the presence of a lunar liquid core modeled in the theory.

5. Terrestrial tidal effects

We present in this section the direct action of the terrestrial tidal bulge characterized by two parameters k_2 (Love number) and ϵ (phase lag) on the Moon's physical librations. A conventional potential Love number $k_2 = 0.3$ for the Earth and a lag angle $\epsilon = 0.0407$ radians have been used. Other values more accurately fitted to the observations could easily be introduced. The expansion of the induced tidal variations of the harmonics coefficients C_{2i} and S_{2i} are given as follows (formula (12) in Bois & Journet (1993) but rectified of some misprints):

$$
\begin{aligned}
\Delta C_{20} &= k_2 \frac{M}{m} \left(\frac{R_\oplus}{r}\right)^3 P_{20}(\sin\delta) \\
\Delta C_{21} &= \frac{1}{3} k_2 \frac{M}{m} \left(\frac{R_\oplus}{r}\right)^3 P_{21}(\sin\delta)\cos(\Theta - \alpha - \epsilon) \\
\Delta S_{21} &= \frac{1}{3} k_2 \frac{M}{m} \left(\frac{R_\oplus}{r}\right)^3 P_{21}(\sin\delta)\sin(\Theta - \alpha - \epsilon) \\
\Delta C_{22} &= \frac{1}{12} k_2 \frac{M}{m} \left(\frac{R_\oplus}{r}\right)^3 P_{22}(\sin\delta)\cos(2(\Theta - \alpha - \epsilon)) \\
\Delta S_{22} &= \frac{1}{12} k_2 \frac{M}{m} \left(\frac{R_\oplus}{r}\right)^3 P_{22}(\sin\delta)\sin(2(\Theta - \alpha - \epsilon))
\end{aligned}
\tag{1}
$$

where r, α, δ are the equatorial coordinates of the Moon and Θ the Greenwich sidereal time (M is the mass of the Earth and $R_\oplus$ its equatorial radius, m is the mass of the Moon). We have assumed that the coefficients k_2 and ϵ are the same for all second degree harmonics. These coefficients are calculated in a terrestrial coordinate system and have to be rotated in a frame parallel to the lunar reference frame. The rotation formulae are given as follows in a general form (Borderies 1978):

$$
\begin{aligned}
P_{lm}(\sin\varphi)e^{im\lambda} = \frac{1}{(l-m)!} \sum_{m'=-l}^{l} (l-m')! E_{lm}^{m'}(\psi, \theta, \phi) \\
\times P_{lm'}(\sin\varphi')e^{im'\lambda'}
\end{aligned}
\tag{2}
$$

with the Eulerian angles ψ, θ, ϕ defined in Sect. 4; $E_{lm}^{m'}$ are the Euler functions defined in Borderies (1978). Afterwards the coefficients are introduced in the expressions of the torques T_{Ox}, T_{Oy}, T_{Oz} of the paper by Schutz (1981) giving the figure-figure effects of rigid bodies (see also the expansion in Bois et al. 1992). The resulting torque is then added to the figure-figure torque already present in the theory. Fig. 1 shows the resulting effects plotted over 240 years and produced by only considering the direct torque on the Moon from the tidal bulge on the Earth. As these effects have been obtained as a difference of two numerical integrations, first disregarding the terms due to the tidal bulge acting on the lunar rotation, secondly taking account of them, we call them $(\Delta\psi, \Delta\theta, \Delta\phi)$. They represent a subclass of librations expressing a specific signature. It should be emphasized that the direct differences of Euler angles place only the upper limit for the magnitude of the studied effects with respect to their observability, initial conditions of the integrations not being adjusted. This point does not take place in the present case. The amplitudes are of the order of a few microseconds of arc. They are well below the threshold of precision of the current LLR observations.

However, from a qualitative point of view, the θ nutation angle presents in fact a long period libration motion (with respect to first results in Bois & Journet 1993). The period of 80.1 years refers to the second resonant frequency related to the synchronous spin-orbit motion of the Moon around a point-like Earth (the first one being 2.9 years). It is here excited under the

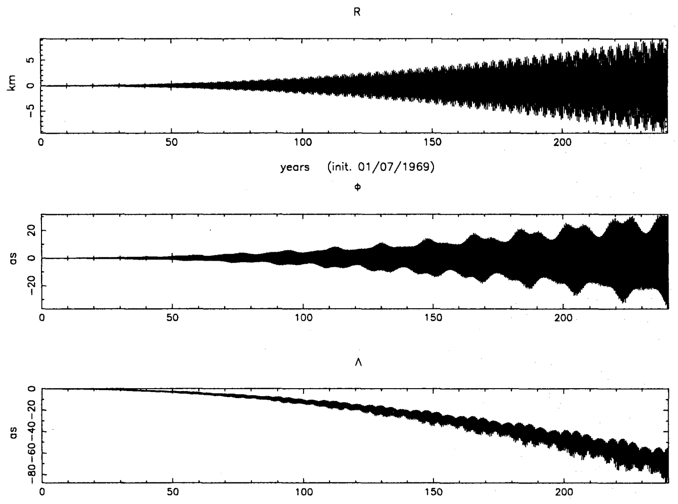

Fig. 3. Secular variation of the lunar longitude as it appears in the Moon's orbital motion plotted in geocentric spherical coordinates (r, Φ, Λ), and obtained by considering the action of the terrestrial bulge advance on the lunar orbital motion. Arcseconds are on the vertical axis (but R in km) and years on the horizontal axis. The secular effect in the Λ orbital longitude gives the usual value for the lunar orbit's secular acceleration: $\dot{n} \simeq -25''/cy^2$.

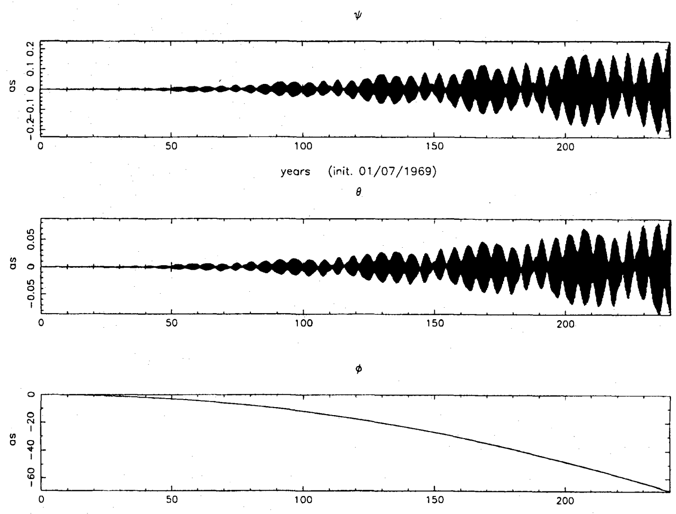

Fig. 4. Secular variation of the Moon's rotation rate, represented by the Euler angles characterizing the lunar rotation, and obtained by considering the indirect effect of the advance of the terrestrial tidal bulge on the lunar spin via its effect on the lunar longitude. Arcseconds are on the vertical axis and years on the horizontal axis. The lunar rotation is accelerated by the same amount as the lunar orbital longitude preserving the synchronous rotation (cf. Fig. 3). The ϕ angle of lunar proper rotation permits to infer the secular slowing down of the lunar rotation: $\dot{\omega}/\omega \simeq -14.10^{-9}\, cy^{-1}$.

gravitational action of the Earth's tidal bulge according to the behaviour of Fig. 1. Let us note that the resonant frequency of 2.9 years is clearly visible in the ϕ angle as already shown in Bois & Journet (1993). Two phenomena contribute to these librations: the variation of the Earth's gravity gradient due to the bulge formation, and the advance of the bulge with respect to the Earth–Moon direction. Fig. 2 shows the specific contribution of the terrestrial phase lag obtained by keeping k_2 fixed and varying ϵ in (1) between the two integrations. The global behaviour is the same, with the same periods, but with a phase difference in time and with amplitudes at least one order of magnitude lower. The advance of the terrestrial tidal bulge modifies only slightly the modulation.

6. Secular variation of the Moon's rotation rate

The tides raised by the Moon on the Earth appear as a bulge leading the Earth-Moon line by a phase lag ϵ. The resulting geocentric acceleration of the Moon has been computed using the following formula (Williams et al. 1978, Newhall et al. 1983) :

$$\ddot{\mathbf{r}}_{tides} = -\left(\frac{3k_2Gm}{r^3}\right)\left(1+\frac{m}{M}\right)\left(\frac{R_\oplus}{r}\right)^5 \begin{cases} x + y\epsilon \\ y - x\epsilon \\ z \end{cases} \qquad (3)$$

where G is the gravitational constant, m the mass of the Moon, M the mass of the Earth and $R_\oplus$ its equatorial radius. (x, y, z) are the components of the geocentric position vector $\mathbf{r}$ of the Moon with z parallel to the Earth's spin axis. Fig. 3 then presents the effects of the terrestrial bulge advance on the Moon's orbital motion plotted in geocentric spherical coordinates r, Φ, Λ. The effects are obtained by keeping k_2 fixed and varying ϵ in (3) between two integrations of the whole spin-orbit system. The secular part in the fast variable Λ (orbital longitude) gives the usual value of the lunar secular acceleration in orbital mean longitude, namely $\dot{n} \simeq -25''/cy^2$, in good agreement with Williams et al. (1978). Fig. 4 presents, as a consequence, the indirect effect of ϵ in (3) on the lunar spin via its direct effect on the orbit, keeping in both integrations the same direct actions on the rotation. Computations have been performed by using the simultaneous integration of the coupled spin-orbit equations. This expresses a direct spin-orbit interaction (indirect effect from the orbit perturbations on the rotation). This interaction excited by the Moon's spin-orbit resonance 1/1 and related to the terrestrial bulge advance is characterized by a secular acceleration of the Moon's rotation rate, as shown in Fig. 4. This mechanism of secular variation of the Moon's rotation rate, which is a slowing down, follows the mechanism of secular lengthening of its semi-major axis. On average indeed, torques on the lunar figure keep one face of the Moon toward the Earth. This synchronous rotation is preserved when considering the accelerations of both the lunar orbit and rotation due to moon-raised tides on the Earth. Averaged over long times, the lunar rotation is accelerated by the same amount as the lunar orbital longitude. The synchronism is then retained :

$$\dot{n} \simeq \dot{\omega} \simeq -25''/cy^2 \quad \text{and} \quad \dot{\omega}/\omega \simeq -14.10^{-9}\, cy^{-1}.$$

With this value of $\dot{n}$, let us recall that we can infer a tidal $\dot{\omega}/\omega \simeq -27.10^{-9}\, cy^{-1}$ for the Earth, whereas there is also a positive acceleration in the Earth's rotation rate of a non tidal origin superimposed on the tidal secular decrease $\dot{\omega}/\omega \simeq +7.10^{-9}\, cy^{-1}$ (Lambeck 1980; Mignard 1986).

7. Conclusion

The direct effect of the terrestrial tidal bulge on the Moon's physical librations as well as the contribution due to the bulge advance have been isolated from the expansion of the related torque acting on the lunar rotation. These effects present a long period of libration (80 years) with very small amplitudes. The direct action of the advance of this terrestrial tidal bulge on the Moon's orbital motion has been also computed as well as its indirect effect on the lunar rotation leading to isolate a spin-orbit interaction. Computations have been performed by using the simultaneous integration of the spin-orbit equations including consequently the coupling effects related to the resonance state of the Moon. This interaction permits to account for the maintaining of the Moon's synchronous rotation. The mechanism of the secular acceleration of the lunar longitude is matched by the secular acceleration of the lunar rotation ($\dot{n} \simeq \dot{\omega} \simeq -25''/cy^2$ and $\dot{\omega}/\omega \simeq -14.10^{-9}\, cy^{-1}$).

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