Polynomial approximations to Delta T, 1620–2000 AD

Jean Meeus & Larry Simons

The difference between uniform Dynamical Time and Universal Time, known as ΔT (Delta T), is needed in several astronomical calculations, such as eclipses and occultations, or to reduce measured positions of minor planets to a uniform time scale for the aim of orbital determinations. As Universal Time is based on the variable rotation of the Earth, the quantity ΔT reflects the irregularities of that rotation, so it varies slowly but rather irregularly with time – see Figure 1.

We have done some work on empirical formulae for approximating ΔT and have derived a set of polynomials of the 4th order which are accurate to within ±1 second for the period 1620-2000. Our results are given in Table 1. These polynomials can easily be implemented in a calculation program, thus avoiding the use of a table or a set of data.

The formula to be used is

$$\Delta T = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4$$

which can be written as

$$\Delta T = a_0 + u[a_1 + u(a_2 + u(a_3 + u a_4))]$$

This way of expressing a polynomial is called Horner's method, an approach especially well suited for programming because powers are avoided.

The quantity u is equal to

$$u = k + (\text{year} - 2000.0)/100,$$

where 'year' may be used with decimals, for instance 1971.5 meaning the middle of the year 1971.

In other words, the quantity u is measured, in centuries, from the middle of the time interval mentioned in the first column of Table 1. The purpose of the k quantities is simply to make the independent variable u as small as possible during that interval.

Table 1. Polynomial expressions, 1620 to 2000.

Time interval (years)	k	a_0	a_1	a_2	a_3	a_4
1620 to 1690	3.45	40.3	−107.0	50	−454	1244
1690 to 1770	2.70	10.2	11.3	−1	−16	70
1770 to 1820	2.05	14.7	−18.8	−22	173	6
1820 to 1870	1.55	5.7	12.7	111	−534	−1654
1870 to 1900	1.15	−5.8	−14.6	27	101	8234
1900 to 1940	0.80	21.4	67.0	−443	19	4441
1940 to 1990	0.35	36.2	74.0	189	−140	−1883
1990 to 2000	0.05	60.8	82.0	−188	−5034	0

If, instead of the year, the Julian Day is used, then

$$u = k + (\text{JD} - 2451545.0)/36525$$

or, if T is the time in Julian centuries from the beginning of AD 2000.0 (negative before 2000.0),

$$u = k + T.$$

It is important to note that each expression is valid only for the period mentioned in the first column of the table. If the first polynomial were used for the epoch 1700.0, we would obtain 11.9 seconds (instead of 7.5); the same expression would give 89 seconds (instead of 10) for 1720.0, and 4105 seconds for 1800!

Example: Epoch 1971.5. We use the seventh expression, $k = 0.35$, $a_0 = 36.2$, etc., and we find $u = +0.065$, whence $\Delta T = 41.74$ seconds.

The expressions given in Table 1 represent the results of a lot of trial and error experimentation (mainly by author LS) to find the best points for the 'knots', the extreme years for each time interval. For instance, why does the fifth polynomial cover only 30 years, from 1870 to 1900, while the fourth one covers 50 years? When we tried a least-squares fit through the points 1870 to 1910, we found that a polynomial of the 4th degree would yield a maximum error of 1.4 seconds. Therefore we were forced to consider the shorter period 1870–1900.

In Figure 1, the vertical dashed lines correspond to the knots.

The data used for our analysis were taken from the following sources: for the years earlier than 1987, pages K8 and K9 of the *Astronomical Almanac* for 1988; from 1987 on, the *Bulletins* issued by the International Earth Rotation Service (IERS), Paris. However, the

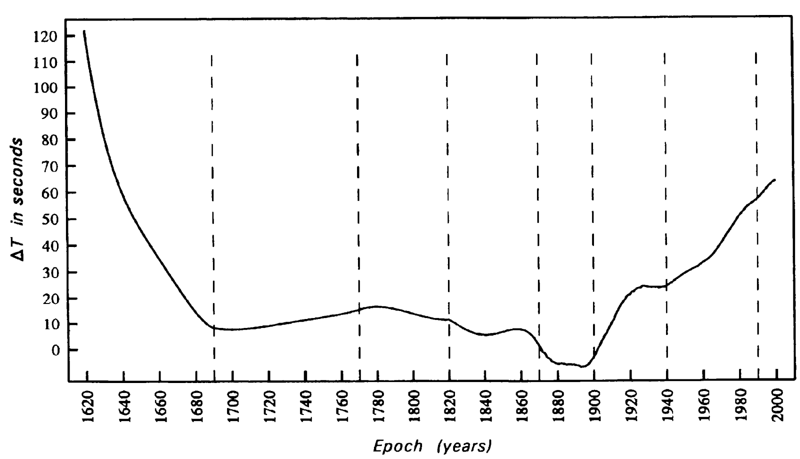

Figure 1. Variation of ΔT with epoch, 1620–2000.

Table 2. Instants when a leap second was introduced, 1972 to 1999.

1972 Jul 1.0	1983 Jul 1.0
1973 Jan 1.0	1985 Jul 1.0
1974 Jan 1.0	1988 Jan 1.0
1975 Jan 1.0	1990 Jan 1.0
1976 Jan 1.0	1991 Jan 1.0
1977 Jan 1.0	1992 Jul 1.0
1978 Jan 1.0	1993 Jul 1.0
1979 Jan 1.0	1994 Jul 1.0
1980 Jan 1.0	1996 Jan 1.0
1981 Jul 1.0	1997 Jul 1.0
1982 Jul 1.0	1999 Jan 1.0

values earlier than 1955 have been corrected by the quantity -0.00002388 (year – 1955)^2 seconds, based on Chapront's new value $-25.7376''$/century^2 (instead of -26) for the tidal acceleration of the Moon.

After AD 2000

Let us again insist on the fact that the last expression in Table 1 should not be used for years after 2000. As is evident from Figure 1, ΔT is not varying regularly with time. In the long run, ΔT should increase more and more rapidly the farther we go into the future, since the rotation of the Earth is generally slowing down. However, while during the years 1968 to 1980 the increase was almost exactly one second per year, from 1997 June 1 to 1998 June 1 it was only 0.65 second, and during the next twelve months it was only 0.38 second, which means that during the last years the rotation of the Earth was temporarily *speeding up*! Will the yearly increase of ΔT continue to decrease? When will it finally start increasing again?

All we can do for the near future is to extrapolate 'intelligently', although it may be just a guess. We may propose a value of 70 seconds for AD 2010, and 80 seconds for 2020.

Dynamical Time and UTC

In what precedes, we considered the difference ΔT between Dynamical Time and 'true' Universal Time. But there exists another time scale known as UTC, or Coordinated Universal Time. UTC is only an approximation to UT, being maintained within 0.9 second of it by the introduction of one-second steps, the so-called leap seconds, when necessary. UTC is an atomic time scale distributed by the broadcast time services. In the interval between two successive leap seconds, the difference Dynamical Time minus UTC remains exactly constant.

While UT itself and hence ΔT are needed in several calculations, for instance local circumstances of occultations and solar eclipses, the difference between Dynamical Time and UTC (not UT) may be needed in other cases, for example when observations have been timed using broadcast time signals.

The difference between Dynamical Time and UTC is equal to 42.18 seconds + the number of leap seconds since the beginning of 1972. Table 2 lists all the leap seconds until the end of 1999.

Addresses: Jean Meeus, Heuvestraat 31, B-3071 Erps-Kwerps, Belgium
Larry Simons, PO Box 12914, London N12 7EB, UK

Received 1999 July 3; accepted 1999 September 1