THE ASTRONOMICAL JOURNAL VOLUME 75, NUMBER 5 JUNE 1970

The Secular Acceleration of the Moon

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(Received 30 March 1970)

If we assume that Atomic Time is a uniform time scale over a few decades, then we may analyze the behavior of the mean longitude of the moon against that time scale. Previously, the moon's mean longitude was used to determine Ephemeris Time; and systematic trends were undetectable, being completely absorbed into the time scale. An extensive analysis of occultations of stars by the moon between 1955 and 1969, since Atomic Time has been available, indicates a residual secular acceleration of $(-15''\pm2'')\ T^2$ (T in centuries), in addition to the previously assumed $-11''.22T^2$ tidal acceleration determined by Spencer-Jones, and the $+7''.14T^2$ dynamical acceleration determined by Brown. Possible causes of this residual acceleration include: (i) additional tidal acceleration; (ii) long-period deficiencies in the lunar theory; (iii) secular variation in length of the Atomic second relative to the Ephemeris second. Analysis of possible sources of systematic error affecting this result seems to imply that the total acceleration not yet accounted for theoretically (including the tidal acceleration) lies between $-18''\ T^2$ and $-34''\ T^2$. This does not exclude the possibility that the acceleration has remained constant over the past 30 centuries, as judged by recent results from studies of ancient observations.

I. BACKGROUND

A SECULAR acceleration of the moon's mean longitude of $+10''\ T^2$ (T in centuries) was detected observationally well before 1800. In 1787, Laplace showed that this acceleration was a consequence of the secular decrease in the eccentricity of the Earth's orbit around the sun, which in turn is due to planetary perturbations. This effect will be referred to here as the "planetary" acceleration of the moon.

Over 60 years later, J. C. Adams (1853) showed that when higher-order terms were included in the calculations, the planetary acceleration was only about $+6''\ T^2$. The discrepancy between theory and observation was made still larger when Newcomb (1878) derived a new value for the acceleration from studies of ancient eclipse records.

Spencer-Jones (1939) derived the apparent secular accelerations of the sun and moon, affected by the variable rotation rate of the Earth, which together form the basis of the currently accepted value for the moon's secular acceleration. This current value consists of a "dynamical" part and a "tidal" part. The dynamical acceleration, $+7''.14T^2$ at 1900.0, is a combination of the planetary acceleration of $+6''.03\ T^2$ as computed by Brown (1915), and an acceleration due to a secular change in general precession of $+1''.11\ T^2$, determined by Newcomb (1906). The tidal acceleration, $-11''.22\ T^2$, was determined by Clemence (1948) by combining the solar and lunar accelerations of Spencer-Jones so as to remove the effect of the variability of the Earth's rotation. The formal probable error is only $\pm0''.53\ T^2$, but possible sources of systematic error are numerous. In particular, corrections applied to the observations to smooth out the effects of changes in observing procedure are sufficient by themselves to alter the derived acceleration by 100%. The association of this acceleration with the effect of tidal friction on the moon was purely for want of a better hypothesis.

II. NEW INVESTIGATION

A discussion of 7000 observations of occultations of stars by the moon between 1950 and 1969 has recently been completed at the U. S. Naval Observatory (Van Flandern 1969). For the observations since 1955, Atomic Time was utilized in the reduction rather than Ephemeris Time, since the former is independent of the moon's motion. The discussion also included corrections for limb irregularities (Watts 1963), star positions reduced to the FK4 coordinate system, the lunar ephemeris designated $j=2$ (Supplement 1968), and other refinements.

In all, nearly 40 parameters were investigated in the discussion. Of particular interest here is the result for the secular acceleration of the moon's mean longitude, which is now just detectable in the observations covering slightly over 14 years of Atomic Time data. The marginal sensitivity of the observations to this parameter over so short a time can be appreciated by noting that, measured from an epoch near 1962.0, T^2 never exceeds 0.005 for the period in question. However, adding just six more years of observations will double the sensitivity of the observations to determining the acceleration; and the situation will continue to improve as T^2 with time.

The result of the present discussion is a residual acceleration of $(-15''\pm2'')\ T^2$, in addition to the tidal and dynamical accelerations already mentioned. Or, regarding only the dynamical acceleration as firmly established on a theoretical basis, the total acceleration not yet accounted for theoretically is $(-26''\pm2'')\ T^2$. Possible causes of this acceleration may include: (i) tidal friction; (ii) secular change in the length of the Atomic second relative to the Ephemeris second; (iii) some long-period (30–100 yr) deficiencies in the lunar theory, probably the planetary part, which would have the appearances of an acceleration over the 14 years covered by this discussion.

The quoted probable error of the result is the formal error from the solution, and does not include the influence of possible systematic errors. Many sources of systematic error were considered, and the discussion was repeated several times to investigate the effects of each. Among these, the following were found to have some influence on the result: (i) difference between UT1 and UTC; (ii) datum corrections to the geodetic coordinates of the observers; (iii) personal equation (reaction time) in the visual occultation observations; (iv) distribution of visual observations relative to photoelectric observations; (v) nonuniformity of distribution of all observations; (vi) correlations with various other solution parameters; (vii) influence of including pre-1955 observations with an extrapolted Atomic Time scale.

After considering uncertainties resulting from each of the above causes, a much larger probable error than the formal one was indicated. However, it is difficult to see how any combination of these known sources of error could produce a result for the total residual secular acceleration (not yet accounted for theoretically) outside the range $-18'' T^2$ to $-34'' T^2$, at the epoch 1962.0. Hence, a large correction is implied to the value formerly assumed.

A recent study of the secular acceleration of the moon from ancient observations (Newton 1969) resulted in estimates of $(-20''.8 \pm 2''.2)\ T^2$ near the year 200 B.C., and $(-21''.2 \pm 3''.1)\ T^2$ near 1000 A.D. Hence, the new result for the current secular acceleration of $(-26'' \pm 8'')\ T^2$ is not inconsistent with the hypothesis that the value has remained nearly constant over the last 30 centuries.

III. CONCLUSION

The secular acceleration of the moon over the period 1955–1969, as determined from occulatation observations, differs substantially from the value assumed in current lunar theories. Whether this discordance is due to some real physical cause, and may be extrapolated to past and future times, or whether the discordance is due to a defect in the lunar theory or some other source of systematic error, and is therefore not representative of the actual acceleration, will be clearer after a few more years of observations become available.

ACKNOWLEDGMENTS

I would like to thank Dr. Raynor L. Duncombe for a critical review of this discussion and for many helpful suggestions.

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