THE SECULAR ACCELERATIONS OF THE MOON'S ORBITAL MOTION AND THE EARTH'S ROTATION*

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Abstract. The results and methods of determining the secular accelerations of the Moon's orbital motion and the Earth's rotation from astronomical observations are critically reviewed. In particular, the effect on these results is considered should Spencer Jones' value for the secular acceleration of the Moon be revised. General relationships are deduced between these accelerations, the rate of dissipation of energy in the Earth and the fractional change in the rate of rotation of the Earth. It is shown that the theory of tidal torques alone does not completely account for *any* of the wide range of results for the retardation of the Earth deduced from astronomical observations.

1. Introduction

It is very appropriate that we should discuss this classical subject at this time and place, since it was on these very islands some 2000 years ago, during the flowering of a great civilization, that some of the crucial observations were recorded on which virtually the whole subject rests. My intention in contributing to this wide-ranging subject, already well covered in three books: *The Earth* (Jeffreys, 1952, 3rd Edition); *The Rotation of the Earth* (Munk and MacDonald, 1960); and *The Earth-Moon System* (Eds. Marsden and Cameron, 1966) is not to attempt to set out particularly original ideas, but rather to review the results and methods of deduction already made from observations. In particular, I will show the implications which follow from the possible revision of Spencer Jones' determination of the current value for the secular acceleration of the Moon. Too much reliance can be placed on Spencer Jones' determination, and I have already attempted (Morrison, 1971) to demonstrate how the standard error of his solution should at least be doubled. The recent renewal of interest in the subject stems partly from the fact that it may soon be possible to derive a reliable value for the current secular acceleration of the Moon from the analysis of some 30000 lunar occultations made since 1955.5, when the atomic-time scale was introduced, thus enabling us to remove accurately the effect of irregular fluctuations in the Earth's rotation from the observations. Before the advent of atomic clocks, two methods had principally been used for separating the Moon's secular acceleration from the Earth's variable rate of rotation:

(1) using ancient (circa A.D. 0) records of total solar eclipses and other astronomical phenomena;

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(2) using modern (1700–1940) telescopic observations of the positions of the Moon and Sun and timings of the transits of Mercury across the Sun's disc.

I will first discuss the methods and deductions made from the ancient data, confining myself to the total solar eclipses which probably constitute the most accurate and reliable data. However, before doing so, I define some basic quantities and concepts.

The independent variable in the gravitational theories of the Sun, Moon and planets is Ephemeris Time (ET). It is regarded as a uniform, dynamical time scale, independent of the rotation of the Earth and is denoted by t; the unit is generally the Julian century of 36525 ephemeris days (cy). Derivatives (dotted) are usually formed with respect to t unless otherwise stated.

The observations discussed in this paper were necessarily referred to the non-uniform time scale, Universal Time (UT), denoted by T. The cumulative difference between ET and UT since A.D. 0 is several hours.

The 'secular acceleration of the Moon' with respect to t is denoted by $\dot{n}$; it has the value of twice the coefficient of t^2 in the expression for the mean longitude in the lunar theory after the removal of the secular part due to gravitational theory; in other words, it is regarded as arising from tidal torques operating in the Earth-Moon system. The 'secular accelerations of the Sun and Moon' with respect to T are here denoted by $2L'$ and $2L$, respectively (although Fotheringham and others referred to L' and L as the secular accelerations). They have the value of twice the coefficient of T^2 in the expressions for the mean longitudes when expressed in Universal Time.

2. Ancient Solar Eclipses

At far distant epochs (say -20 cy) the differences (F and F') in the Moon's and Sun's mean longitudes between those deduced from observations (in UT) and those calculated from modern theories (in ET) can be almost wholly attributed to the effect of the secular accelerations $2L$ and $2L'$; the errors arising from linear terms are much smaller. Thus:

$$\begin{aligned} 2L &= 2F/(\text{interval from 1900 in cy})^2 \\ 2L' &= 2F'/(\text{interval from 1900 in cy})^2 . \end{aligned} \tag{1}$$

The observed occurrence of an identifiable total solar eclipse at a known site provides the difference in position on the Earth's surface between the observation site and the calculated path of totality. This leads to a linear relationship between L and L';

$$L' = aL + b . \tag{2}$$

Ideally, the locus of intersection of such lines for eclipses of similar epoch establishes the values of L' and L. In practice, however, many eclipse records are vague and ambiguous in interpretation, so that Fotheringham (1920) plotted two lines for each eclipse on the (L, L') plane, defining the extreme limiting values of a and b consistent with the records. I have reproduced his famous diagram in Figure 1 with his absissa reduced by $6''.1/\text{cy}^2$ which removes the purely gravitational part of the Moon's secular

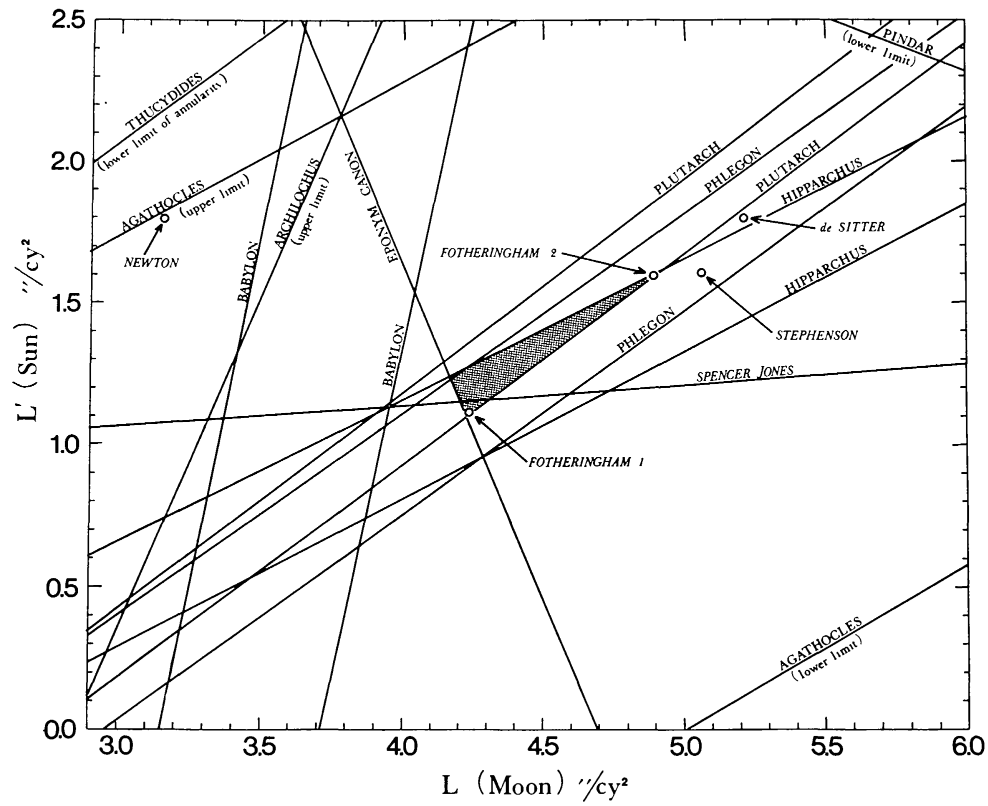

Fig. 1. Fotheringham's diagram for ancient solar eclipses with Spencer Jones' result from modern telescopic observations.

acceleration in longitude arising from planetary perturbations. The shaded area in Figure 1 defines the area of maximum likelihood for the values of L' and L.

The 'secular acceleration of the Sun L'', as Fotheringham referred to it, is interpreted as the retardation of the Earth's rate of rotation with respect to the uniform time scale of ephemeris time. The secular acceleration of the Moon, $\dot{n}$, with respect to ephemeris time is therefore given by

$$\tfrac{1}{2}\dot{n} = L - n/n' \cdot L' \, (''/\text{cy}^2) \quad (n/n' = 13.37) \tag{3}$$

where n and n' are the mean motions of the Moon and Sun. For use later, we express $2L'\,(''/\text{cy}^2)$ in terms of the fractional change of the rate of rotation of the Earth, ω, measured in $''/\text{cy}$. The mean motion of the Sun is approximately $0\overset{''}{.}130 \times 10^7$ per year, therefore

$$2L' = -\,0.130 \times 10^9 \,(\dot{\omega}/\omega)\,(''/\text{cy}^2). \tag{4}$$

And using Equation (3), we find

$$2L = \dot{n} - 1.74 \times 10^9 \,(\dot{\omega}/\omega)\,(''/\text{cy}^2). \tag{5}$$

Fotheringham did not deduce particular values of L and L' from total solar eclipses alone, but the two points in Figure 1 correspond to (4.1, 1.1) and (4.9, 1.6), giving $\tfrac{1}{2}\dot{n}\,(''/\text{cy}^2)$ equal to -10.5 and -16.5 respectively. De Sitter (1927) in a rediscussion of

Fotheringham's work deduced (5.22±0.30, 1.80±0.16), giving $\frac{1}{2}\dot{n}=18.8\pm2.1$ (p.e.). Stephenson (1971) has recently undertaken a similar study to Fotheringham's by adding discussions of oriental material, and he finds the solution (5.07±0.25, 1.61±0.17), giving $\frac{1}{2}\dot{n}=-16.5\pm2.3$ (p.e.). Using a statistical analysis, rather than Fotheringham's diagrammatical approach, Newton (1970) found (3.16±1.3, 1.80±0.22)* for large solar eclipses before A.D. 500, corresponding to $\frac{1}{2}\dot{n}=-20.9\pm2.6$ (s.e.). His result is quite incompatible with Fotheringham's area of maximum likelihood.

3. Modern Telescopic Observations

The secular change in the rate of the Earth's rotation cannot be derived from observations over the past 200 years because the effect due to tidal friction is masked by relatively short and erratic changes in the rate of rotation due to other causes; but the secular acceleration of the Moon can be derived by combining observations of the Moon with those of the Sun and Mercury in a method used by de Sitter (1927) and Spencer Jones (1939).

As with ancient observations, we take $F'(T)$ and $F(T)$ to be the differences between the observed and calculated (gravitational) longitudes of the Sun and Moon. Following Murray (1957) we can express $F(T)$ as follows:

$$F(T) = a + bT + \tfrac{1}{2}(\dot{n} + s)\,T^2 + B(T). \tag{6}$$

The coefficient of T^2 consists of two parts: $\dot{n}$, the true acceleration with respect to the uniform time scale of ephemeris time; and s, an estimate of that part of the secular acceleration due to the change in the Earth's rotation. The observed fluctuations in the Moon's longitude due to the non-uniform rotation of the Earth are absorbed in $B(T)$. Note that it is not possible to separate s uniquely from a similar term contained in $B(T)$, but this is not necessary in deriving $\dot{n}$. For the Sun observations we have

$$F'(T) = a' + b'T + \tfrac{1}{2}(n'/n)\,sT^2 + (n'/n)\,B(T) \tag{7}$$

neglecting $\dot{n}'$, which is very small. Differentiating (6) and (7) twice with respect to T, and assuming that $\dot{n}$ and s are constant, we obtain

$$\ddot{F}(T) = \dot{n} + s + \ddot{B}(T), \tag{8}$$

and

$$(n/n')\,\ddot{F}'(T) = \qquad s + \ddot{B}(T). \tag{9}$$

Subtracting:

$$\ddot{F}(T) - (n/n')\,\ddot{F}'(T) = \dot{n}. \tag{10}$$

Therefore, $\dot{n}$ is derived by fitting a parabola through the differences of the observed discrepancies in the longitudes of the Moon and Sun (or Mercury), the latter reduced to the orbit of the Moon.

* By substitution of his values $-41.8\pm4.3''/\mathrm{cy}^2$ and $-27.7\pm3.4\times10^{-9}/\mathrm{cy}$ for $\dot{n}$ and $\dot{\omega}/\omega$ in Equations (4) and (5).

Spencer Jones put

$$\tfrac{1}{2}(\dot{n} + s) = c$$

and

$$\tfrac{1}{2}(n'/n)\, s = c' \text{ in Equations (6) and (7).*}$$

Thus, recalling Equation (3), we have

$$\tfrac{1}{2}\dot{n} = c - (n/n')\, c' = L - (n/n')\, L'. \tag{11}$$

Note that this is not an identity. In practice, the value of $\dot{n}$ was derived by Spencer Jones by adopting $c = 5.22''/\text{cy}^2$ (de Sitter's value for L; but $c \neq L$), and using occultation observations to give $F(T)$ in Equation (6). Values of $B(T)$ were then derived from this equation after fitting the data to determine the arbitrary constants a and b. The values of $B(T)$ were then substituted in Equation (7), and a', b' and c' determined by least squares fit to $F'(T) - (n'/n)\, B(T)$. He found $c' = 1.23 \pm 0.04$(p.e.)$''/\text{cy}^2$ giving

$$\tfrac{1}{2}\dot{n} = L - 13.37L' = -\,11.22 \pm 0.44 \text{ (p.e.) } ''/\text{cy}^2 \tag{12}$$

from Equation (11). The values of L, L', c and c', together with $\frac{1}{2}\dot{n}$ are gathered in Table I.

TABLE I

Values of L, L' and $\frac{1}{2}\dot{n}$ deduced from ancient eclipses

Author		L	L'	$\frac{1}{2}\dot{n}$ ($''/\text{cy}^2$)
Fotheringham	1	4.2	1.1	−10.5
	2	4.9	1.6	−16.5
de Sitter		5.22	1.80	−18.8
Newton		3.16	1.80	−20.9
Stephenson		5.07	1.61	−16.5
		c	c'	$\frac{1}{2}\dot{n}$
Spencer Jones		5.22	1.23	−11.22

4. Combining Results from the Ancient and Modern Observations

The ancient observations afford us values of both the orbital acceleration of the Moon, $\dot{n}$, and the rotation of the Earth, $\dot{\omega}$; the modern observations only give us a value for the former. Both the ancient and modern results for $\dot{n}$ have weaknesses. The question is whether a stronger solution can be found by combining these results. From Table I one can see the disparity of the results for $\dot{n}$ derived from ancient observations and one is tempted to take Spencer Jones' modern value with its small probable error and combine it with the ancient data to find $\dot{\omega}$. Dicke (1966) and Curott (1966) have adopted this procedure.

* In fact, his notation was slightly different; he substituted 5.22 for $\frac{1}{2}(\dot{n} + s)$ and used c where I have used c'. I use the present notation for consistency where a prime denotes the 'Sun'.

5. Dicke's Solution for L′

Dicke, assuming that the secular acceleration of the Moon has been constant for the last 2000 years, combined Spencer Jones' value for $\frac{1}{2}\dot{n}$ with Fotheringham's results from the analysis of five ancient solar eclipses. Instead of using the limiting lines as shown in Figure 1, Dicke calculated single 'mean' lines for four of Fotheringham's bands and two possible lines for the eclipse of Archilochus corresponding to whether the eclipse was seen at one or other of two islands. From relation (12) he found $13.37L' - 11.22$ and substituted this for L in his equations of the 'mean' lines, which have the

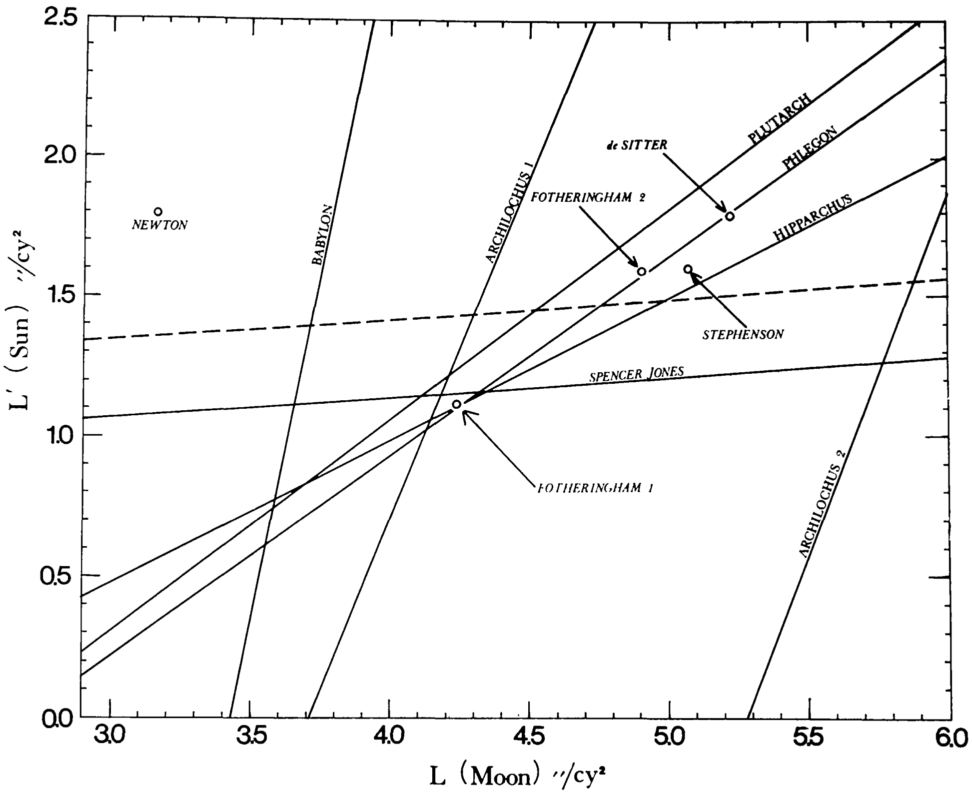

Fig. 2. Diagram showing Dicke's 'mean' eclipse lines and their intersection with Spencer Jones' line. (For dashed line, see text.)

form of Equation (2), and solved for values of L'. I have shown his procedure diagrammatically in Figure 2: the points of intersection of Spencer Jones' line with the eclipse lines give Dicke's values of L' shown in Table II.

From the consistency of the values that he deduced for L', Dicke placed much reliance on his adopted mean value of $1''.17/\text{cy}^2$ and went on to conjecture about the possible change in the gravitational constant, G, over the past 2000 years. However, the consistency of the deduced values of L', using Dicke's method, does not guarantee that they are the best values, because Spencer Jones' line is nearly horizontal in Figure 2 and, therefore, its intersection with the eclipse lines inevitably leads to fairlyc onsistent values of L' (though not of L).

TABLE II

Values of L' ($''$/cy^2) from Figure 2 for different values values of $\dot{n}$

Eclipse	$\frac{1}{2}\dot{n} = -11.22$ (Dicke)	$\frac{1}{2}\dot{n} = -15.0$
Plutarch	1.150	1.464
Phlegon	1.165	1.481
Hipparchus	1.163	1.494
Archilochus 1	1.143	1.435
Archilochus 2	1.258	1.549
Babylon	1.120	1.407
Mean	1.1666	1.472
standard error	0.0195	0.0201

Bearing in mind that a standard error of about $\pm 3''44$/cy^2, rather than the quoted probable error of $\pm 0''.44$/cy^2, might be attached to Spencer Jones' value for $\frac{1}{2}\dot{n}$, it is reasonable to consider a value of, say, $-15''$/cy^2, which is nearer to the 'ancient' determination. I have shown the line

$$L - 13.37L' = -15.0$$

dashed in Figure 2. Its intersections with the eclipse lines give the values of L' (actually calculated rather than read from Figure 2) listed under $\frac{1}{2}\dot{n} = -15.0$ in Table II.

The standard errors of the mean values of L' in Table II are not significantly different.

Curott (1966) employed a basically similar procedure to Dicke. He adopted Spencer Jones' value of $-11''.22$/cy^2 and recomputed the longitudinal displacements on the Earth between the observed and calculated paths of totality for 32 ancient eclipses. Whereas Dicke found a mean value of $1''.17$/cy^2 for L', Curott, in his wider and more thorough investigation, found $1''.1$/cy^2*, with a greater margin of uncertainty. But this method is essentially pre-conditioned to arrive at values of L' near $1''.1$/cy^2.

This may be the best (and only?) procedure for combining the ancient and modern results provided we have a secure value for $\frac{1}{2}\dot{n}$ from the latter. Failing this, we are compelled to 'keep an open mind' on the question, but we can still arrive at some general conclusions about the tidal interaction in the Earth-Moon system without adopting a specific value for $\frac{1}{2}\dot{n}$.

6. The Lunar Tidal Acceleration and the Rate of Dissipation of Energy

Without making the hypothesis that $\dot{n}$ has been constant over the past 2000 years we can arrive at general expressions relating the rate of dissipation of tidal energy, $-\dot{E}$, and the fractional change in the rate of rotation of the Earth due to tidal coupling, $(\dot{\omega}/\omega)_{\text{tidal}}$, with $\dot{n}$.

* By substitution of his result -17×10^{-9}/cy for $\dot{\omega}/\omega$ in relation (4).

Firstly, we establish the relation between -$\dot{E}$ and $\dot{n}$. Following Jeffreys (*The Earth*, p. 218), let the masses of the Earth and Moon be M and m, and let the mean distance between them be r. Let the principal moment of inertia of the Earth about its polar axis be C. Kepler's third law states $n^2r^3 = \text{const}$. Differentiating with respect to time we obtain

$$n\dot{r} = -\tfrac{2}{3}r\dot{n}\,. \tag{13}$$

The angular momentum of the orbital motion of the Moon and Earth about the centre of mass of the two together is

$$\frac{Mmr^2n}{M+m} = \frac{mr^2n}{1.012}, \quad \text{taking} \quad m/M = \frac{1}{81.5},$$

and that of the Earth's rotation about its axis is $C\omega$. By the principle of the conservation of angular momentum, the couple $-N$ acting on the Earth's rotation must be balanced by one of $+N$ tending to increase the orbital angular momentum: that is

$$C\frac{\mathrm{d}}{\mathrm{d}t}(\omega)_{\text{tidal}} = -N \tag{14}$$

and

$$\frac{m}{1.012}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(r^2n) = +N\,. \tag{15}$$

Performing the differentiation of (15) and substituting (13), we find

$$-\frac{mr^2}{3.037}\cdot\dot{n} = +N\,. \tag{16}$$

If N is expressed in ergs, m in g and r in cm, and $\dot{n}$ is in the customary units of $''/\text{cy}^2$, then we require the factor

$$\left(\frac{1}{206\times10^3}\right)\left(\frac{1}{31.6\times10^8}\right)^2 = 4.85\times10^{-25}\,[\text{rad. cy}^2/''\,\text{s}]$$

on the L.H.S. of (16).

The rate at which the lunar couple does work is

$$-\dot{E}_{\text{lunar}} = +N(\omega - n)\,. \tag{17}$$

Substituting (16), we have

$$-\dot{E}_{\text{lunar}} = -(mr^2/3.037)(\omega - n)\,\dot{n}\,. \tag{18}$$

Inserting the following values: $m = 7.35\times10^{25}$ g; $r = 3.84\times10^{10}$ cm; $\omega = 7.29\times10^{-5}$ rad s^{-1}, $n = 0.27\times10^{-5}$ rad s^{-1}; and the factor 4.85×10^{-25}, we find

$$-\dot{E}_{\text{lunar}} = -1.22\times10^{18}\dot{n}\ \text{ergs sec}^{-1}\,. \tag{19}$$

Now there is also a solar torque, N', acting on the oceanic tide, and, according to which model we adopt (see Jeffreys, 8.02 and 8.03), we have $N'/N = 1/5.1$ or $1/3.4$. Therefore,

$$-\dot{E}_{\text{solar}} = \left.\begin{matrix} -0.24 \\ -0.36 \end{matrix}\right\} \times 10^{18}\dot{n} \text{ ergs sec}^{-1}. \quad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{20}$$

The rate of dissipation of tidal energy deduced from the requirement that angular momentum be conserved in the Earth–Moon system is then (after rounding off)

$$-\dot{E}_{\text{tidal}} = \left.\begin{matrix} -1.45 \\ -1.57 \end{matrix}\right\} \times 10^{18}\dot{n} \text{ ergs sec}^{-1}. \quad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{21}$$

I give the results in Table III for $-\dot{E}_{\text{tidal}}$ using the various values of $\frac{1}{2}\dot{n}$ given in Table I. These results are also presented in Figure 3, together with the lines defined by Equation (21).

TABLE III

$-\dot{E}$ derived from values of $\frac{1}{2}\dot{n}$ in Table I

Author		$\frac{1}{2}\dot{n}$ ″/cy²	$-\dot{E}$ N' $N=1/5.1$ 10^{19} erg s^{-1}	$N'/N=1/3.4$
Fotheringham	1	−10.5	3.04	3.30
	2	−16.5	4.78	5.18
de Sitter		−18.8	5.45	5.90
Newton		−20.9	6.06	6.56
Stephenson		−16.5	4.78	5.18
Spencer Jones		−11.22	3.25	3.52

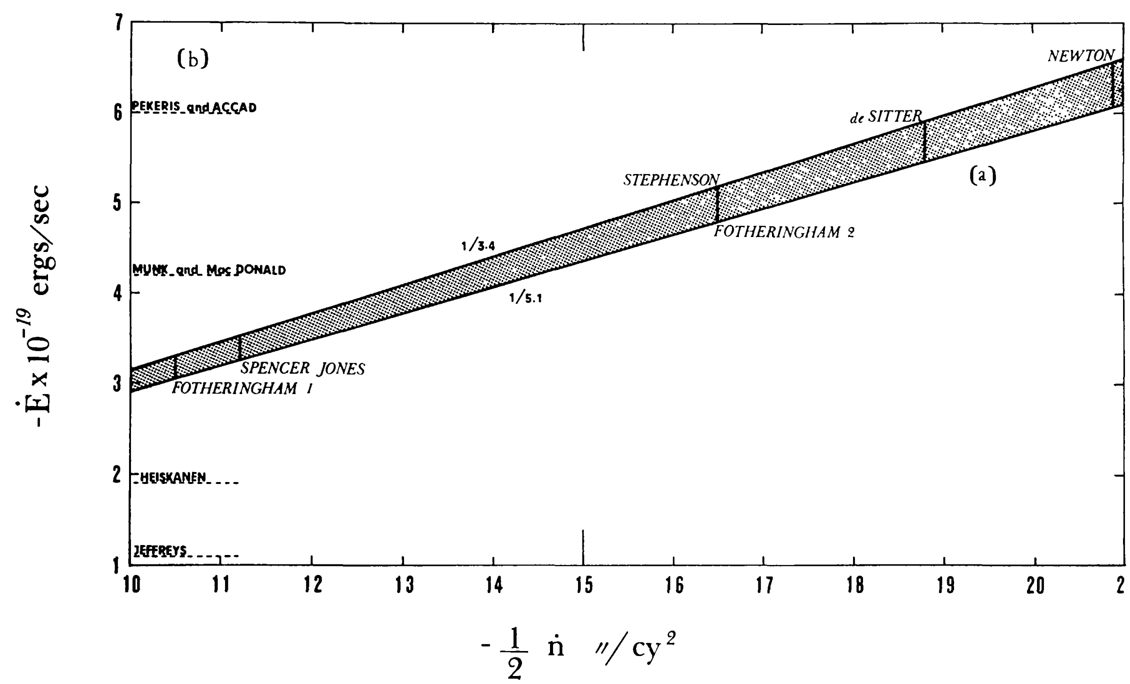

Fig. 3. Rates of energy dissipation in the seas and oceans: (a) deduced from astronomical observations (linear function of $\dot{n}$) and (b) estimated from tidal fluxes.

Estimates of the value of $-\dot{E}_{\text{tidal}}$ have been made independently of the astronomical method given above by studying the flux and bottom friction of the tides in the seas and oceans. The reader is referred to Chapter 11, Section 7 *et seq.* of *The Rotation of the Earth* for a discussion of Jeffreys, Heiskanen, Munk and MacDonald's estimates shown (dashed) in Figure 3. Recently Pekeris and Accad (1969) have solved Laplace's tidal equations numerically for 1° and 2° grids covering the oceans with, and without, a simplified model of tidal friction proportional to a linear velocity law. They found that their theoretical amplitudes for the M_2 tide only agreed with observations after adopting their friction model, and the rate of dissipation of energy which resulted was around 6×10^{19} ergs s^{-1}. Although the values of $\frac{1}{2}\dot{n}$ and $-\dot{E}$ shown in Table III and Figure 3 cover a considerable range, it is probably remarkable that the values deduced from the astronomical data and those from the tides agree to well within a factor of ten. Again, if we could reconcile Spencer Jones' modern value of $\frac{1}{2}\dot{n}$ with a value somewhere near $-15''/\text{cy}^2$, then the astronomical data would give a reasonably consistent pointer for a value of $-\dot{E}$ around 4.5×10^{19} ergs s^{-1}.

7. The Tidal Deceleration of the Earth's Spin Versus the Observed Deceleration

Following from the requirements of the conservation of angular momentum and Kepler's third law, we arrived at relation (14). Differentiating we have

$$(\dot{\omega}/\omega)_{\text{lunar}} = - N/C\omega\,. \tag{22}$$

We take $C = 0.334\ Ma^2$, where a is the radius of the Earth, and substitute the L.H.S. of (16) for N in (22); then

$$(\dot{\omega}/\omega)_{\text{lunar}} = +\,0.986\,(m/M)\,(r/a)^2\,\dot{n}/\omega\,. \tag{23}$$

Taking $m/M = 1/81.5$, $r/a = 60.3$ and $\omega = 47\overset{''}{.}5 \times 10^9/\text{cy}$, we obtain

$$(\dot{\omega}/\omega)_{\text{lunar}} = +\,0.93 \times 10^{-9}\dot{n} \quad /\text{cy}\,. \tag{24}$$

If we take the solar torque, N', according to the ratios given before

$$(\dot{\omega}/\omega)_{\text{solar}} = \left.\begin{matrix} +\,0.18 \\ +\,0.27 \end{matrix}\right\} \times 10^{-9}\dot{n} \quad /\text{cy}\,. \qquad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{25}$$

There is also an acceleration due to the solar torque of -0.30×10^{23} ergs acting on the Earth's atmosphere giving

$$(\dot{\omega}/\omega)_{\text{atmos.}} = +\,1.6 \times 10^{-9} \quad /\text{cy}\,. \tag{26}$$

Adding (24), (25) and (26), we find the total fractional change per century in the rate of rotation of the Earth due to tidal torques is

$$(\dot{\omega}/\omega)_{\text{tidal}} = \left.\begin{matrix} +\,1.11 \\ +\,1.20 \end{matrix}\right\} \times 10^{-9}\cdot\dot{n} + 1.6 \times 10^{-9} \qquad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{27}$$

We substitute (11) for $\dot{n}$ in (27) and find

$$(\dot{\omega}/\omega)_{\text{tidal}} = \left.\begin{matrix} -0.296 \\ -0.320 \end{matrix}\right\} \times L' \left.\begin{matrix} +0.022 \\ +0.024 \end{matrix}\right\} \times L + 0.016 \times 10^{-7} \quad /\text{cy} \quad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{28}$$

If we take the values 4.2 and 5.2 ($''/\text{cy}^2$) for L, we have the fractional change in the Earth's spin per century due to tidal torques as a linear function of L', the apparent acceleration of the Sun seen by an observer on Earth. Equation (4) expresses the units connecting L' and $(\dot{\omega}/\omega)$ which is the *observed* fractional change of spin:

$$(\dot{\omega}/\omega) = -0.154L' \quad \times 10^{-7}/\text{cy}. \tag{29}$$

I show relations (28) and (29) in Figure 4: the difference

$$(\dot{\omega}/\omega) - (\dot{\omega}/\omega)_{\text{tidal}} = \left.\begin{matrix} +0.142 \\ +0.166 \end{matrix}\right\} \times L' \left.\begin{matrix} -0.022 \\ -0.024 \end{matrix}\right\} \times L - 0.016 \quad \times 10^{-7}/\text{cy} \quad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{30}$$

expresses the amount of the fractional *increase* in the Earth's spin due to non-tidal effects. For example, Dicke took $\frac{1}{2}\dot{n} = -11.22$ and found $L' = 1.17$, implying from Equation (12) that $L = 4.42$. Substituting in (30)

$$(\dot{\omega}/\omega) - (\dot{\omega}/\omega)_{\text{tidal}} = \left.\begin{matrix} +5.2 \\ +7.2 \end{matrix}\right\} \times 10^{-9}/\text{cy} \qquad \begin{matrix} 1/5.1 \\ 1/3.4 \end{matrix} \tag{31}$$

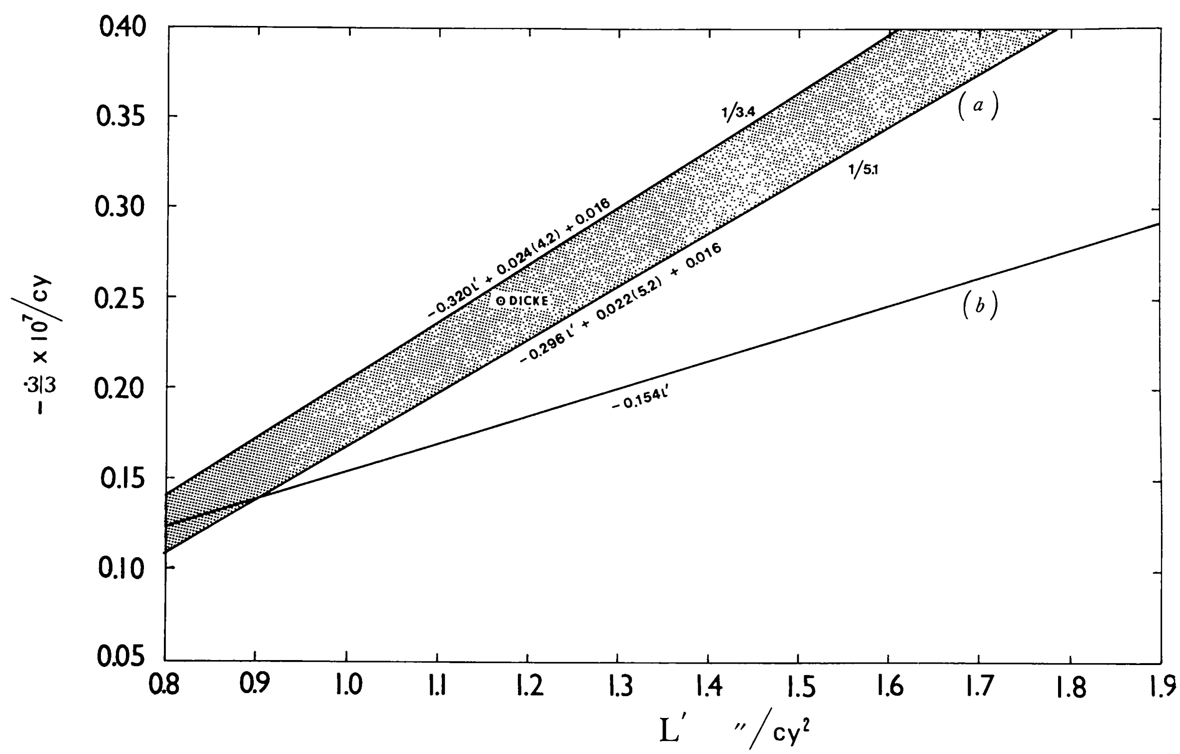

Fig. 4. Fractional decrease per century in the Earth's rate of rotation: (a) $(\dot{\omega}/\omega)_{\text{tidal}}$, deduced from conservation of angular momentum in the Earth-Moon system, and (b) $(\dot{\omega}/\omega)$, the observed change. (For Dicke's point and equations of lines – see text.)

Attributing two-thirds to the 1/3.4 couple, we find a fractional increase of 6.6 parts in 10^9 per century, which is close to Dicke's result of 6.8 parts which includes a small correction for a change in sea level since the time of the eclipses circa A.D. 0. I have shown Dicke's 'two-thirds' value for $(\dot{\omega}/\omega)_{\text{tidal}}$ in Figure 4.

Now if we again conjecture that $\frac{1}{2}\dot{n} = -15.0''/\text{cy}^2$, rather than Spencer Jones' value of -11.22, then L' lies between 1.4 and 1.5 (from relation (11) with $L=4$ and 5) and therefore, from Figure 4, the fractional increase is at least 7 parts, and probably 11 parts in 10^9 per century. Newton's values for L and L' in Table I give an average fractional increase since A.D. 0 of at least 17 parts in 10^9/cy.

8. Concluding Remarks

There are two immediate consequences of revising Spencer Jones' value of $\frac{1}{2}\dot{n}$ from $-11\overset{''}{.}22/\text{cy}^2$ to a value nearer the ancient eclipse results, say, $-15\overset{''}{.}0/\text{cy}^2$:

(1) the rate of dissipation of energy in the Earth is increased (Figure 3); and

(2) the fractional change in the Earth's spin rate is increased (Figure 4).

If tidal torques are the only significant forces operating on the rotation of the Earth, then we expect to 'observe' L' below $0\overset{''}{.}9/\text{cy}^2$ (see Figure 4): we do not. Therefore, some force producing an acceleration is at work, or our value of L' is unreliable. Many authors have searched exhaustively for geophysical forces to produce the required accelerations. The trend of recent investigations using current observations is to find $\frac{1}{2}\dot{n}$ greater (negatively) than $-11\overset{''}{.}22/\text{cy}^2$, giving L' greater than $1\overset{''}{.}1/\text{cy}^2$ (from Equation (3) with $L=3.5$) which strengthens the case for forces producing accelerations.

I now look forward with keen anticipation to see what value of $\frac{1}{2}\dot{n}$ emerges from the analysis of some 30000 occultation observations now under way at the Royal Greenwich Observatory.

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