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Polynomial Approximations for the Correction $\Delta T = E.T. - U.T.$ in the Period 1800-1975 *

by

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ABSTRACT

Polynomial approximations for the empirical difference between *Ephermeris* and *Universal Time* are discussed in order to provide simple arithmetic means for computing purposes. The entire period 1972.6-1978.5 can be covered by a 12th degree polynomial with a mean error of less than one second. All coefficients are significant values well above the 3σ level of the mean coefficient errors. Coefficients, mean errors, and maximal residuals for polynomials of degree 8 to 16 are given as well as least-squares solutions fitting low-degree expressions to some smaller time intervals.

1. Introduction

The rotation of the earth is variable, which causes some fluctuations in the mean solar time. Additionally there exists a secular deceleration due to tidal effects of the earth. The observed positions of all solar system bodies are usually referred to *Universal Time* (*U.T.*). In consequence the observed coordinates of planets and comets deviate from those values which are based on orbit theories. These differences correspond to their respective motions in a time interval equal to the actual error in mean solar time.

The time correction between *U.T.* and *Ephemeris Time* (*E.T.*) is dependent on a fluctuation term B (Spencer Jones 1939) which represents the correction of the sun's tabular mean longitude (Newcomb 1895). The values of B are derived from lunar occultations and meridian observations of major and minor planets. Further details are extensively discus-

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sed in the literature (*e.g.* Clemence 1948, Brouwer 1952, Gondolatsch 1953, Morrison 1973).

This short note aims to present some simple arithmetic expressions for ΔT over the whole period 1800-1975 in order to facilitate the reduction of observing times of planetary bodies. In extension of a recent paper (Bielicki and Ziołkowski 1976), polynomial coefficients for approximations with different aims regarding simplicity and accuracy are given.

2. Results

Fig. 1 contains unsmoothed yearly values of ΔT showing the general trend of this quantity in the period considered. Until 1948.5 values obtained by Brouwer are used. Some additional points for the first half of the nineteenth century are also incorporated (Spencer Jones 1932). For 1949.5 to 1955.5 *B* values derived at U.S. Naval Observatory are used. All later points are from recent volumes of the *American Ephemeris*. One should note, however, that the newest values are extrapolated ones.

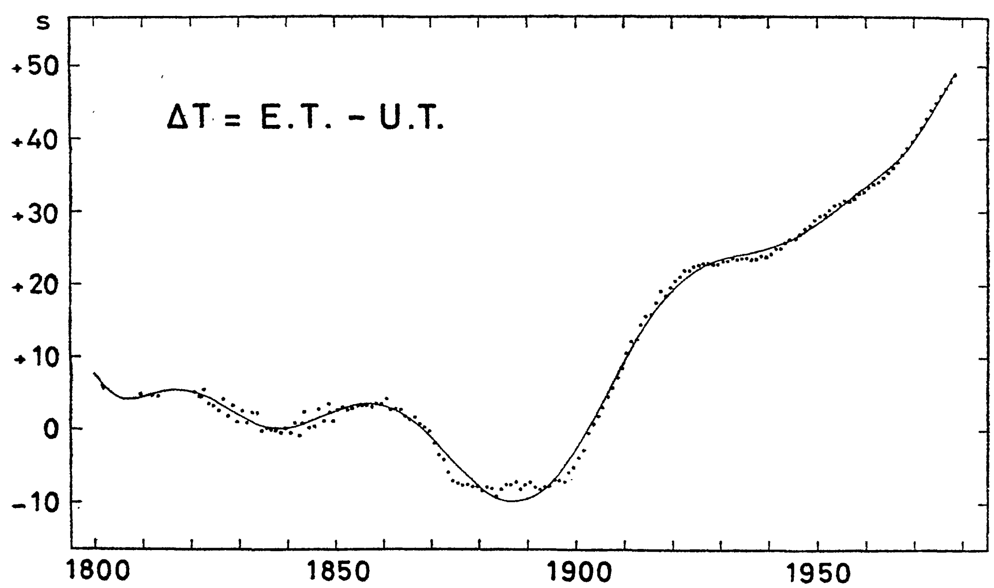

Fig. 1. Unsmoothed values for the difference ΔT 1792.6-1978.5. The solid curve represents the 12th-order least-squares fit mentioned in the text.

Using the whole material we derived least-squares polynomials of degree 8 to 16 (Table 1). The course of the corresponding mean errors as well as that of the maximal residuals shows a remarkable increase of accuracy between degrees 9 and 10 and, respectively, 11 and 12. The 12th-order equation was found to be the lowest one for which the mean error would be less than one second of time. This yields the following expression:

$$\Delta T = -0.000029 + 0.001233\,T + 0.003081\,T^2 - 0.013867\,T^3 - 0.020446\,T^4 + 0.076929\,T^5 + 0.075456\,T^6 - 0.200097\,T^7 - 0.159732\,T^8 + 0.247433\,T^9 + 0.185489\,T^{10} - 0.117389\,T^{11} - 0.089491\,T^{12},$$

where T denotes the time of observation expressed in Julian centuries elapsed since 1900 January 0^d5 *E.T.* This solution is drawn as solid curve in Fig. 1. A time resolution of 10^{-5} days should meet all requirements of planetary work.

Table 1

Coefficients (ν), polynomial degree (N), and errors for some approximations of ΔT in 1800-1975

ν \ N	8	9	10	11	12	13	14	15	16
m.e.	2^{s}31	2^{s}18	1^{s}31	1^{s}27	0^{s}94	0^{s}92	0^{s}91	0^{s}90	0^{s}80
max.	7^{s}43	3^{s}80	4^{s}23	2^{s}94	2^{s}76	2^{s}76	3^{s}37	2^{s}76	2^{s}33
0	−0.000014	−0.000023	−0.000026	−0.000030	−0.000029	−0.000032	−0.000032	−0.000034	−0.000036
1	+0.000880	+0.000852	+0.001094	+0.001092	+0.001233	+0.001241	+0.001205	+0.001216	+0.001128
2	+0.002060	+0.002560	+0.002829	+0.003163	+0.003081	+0.003363	+0.003446	+0.003710	+0.004024
3	−0.004791	−0.004199	−0.009523	−0.009395	−0.013867	−0.014123	−0.012567	−0.013052	−0.007956
4	−0.008312	−0.012464	−0.016433	−0.020729	−0.020446	−0.025586	−0.027042	−0.033607	−0.041579
5	+0.011839	+0.008078	+0.039207	+0.037286	+0.076929	+0.078462	+0.059047	+0.064710	−0.020909
6	+0.015486	+0.026349	+0.048194	+0.067463	+0.075456	+0.109922	+0.117054	+0.178577	+0.246465
7	−0.008881	+0.000349	−0.066624	−0.056195	−0.200097	−0.198521	−0.093995	−0.116051	+0.522865
8	−0.010014	−0.018831	−0.067937	−0.103214	−0.159732	−0.264914	−0.266396	−0.543037	−0.757982
9		−0.007601	+0.040310	+0.017813	+0.247433	+0.217103	−0.058647	−0.052653	−2.519039
10			+0.037832	+0.060474	+0.185489	+0.334511	+0.268018	+0.911623	+0.952888
11				+0.016505	−0.117389	−0.046785	+0.305115	+0.451931	+5.588855
12					−0.089491	−0.169196	−0.015466	−0.763433	+0.391189
13						−0.049379	−0.223300	−0.538684	−6.024034
14							−0.102638	+0.241059	−2.027234
15								+0.197336	+2.553876
16									+1.327078

Table 2

Low-degree polynomial approximations of ΔT.

Period	N	m.e.	max.	$\nu = 0$	1	2	3	4	5
1956.5 - 1978.5	3	0^{s}17	0^{s}26	+0.003472	−0.013912	+0.019758	−0.008598		
1898.5 - 1956.5	5	0.31	0.69	−0.000049	+0.001176	+0.009877	−0.067857	+0.140646	−0.095401
1879.5 - 1898.5	2	0.51	0.95	−0.000073	+0.000248	+0.000695			
1820.5 - 1879.5	5	0.80	1.81	+0.001109	+0.017719	+0.092852	+0.214418	+0.226799	+0.089787
1792.6 - 1820.5	2	0.20	0.26	+0.000553	+0.001159	+0.000676			

Efforts are made in order to derive some very short expressions for certain parts of the entire interval. The coefficients of Table 2 are constructed to reach accuracies well under 10^{-5} days.

It should be noted that all coefficients of all polynomials are well above the 3σ confidence level.

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