

# Changes in the Earth's rate of rotation between A.D. 1672 and 1806 as deduced from solar eclipse timings

F.R. Stephenson<sup>1</sup> and J.H. Lieske<sup>2</sup>

<sup>1</sup> Department of Physics, University of Durham, DH1 3LE, UK

<sup>2</sup> Jet Propulsion Laboratory, Pasadena, CA 91109, USA

Received August 31, accepted December 28, 1987

**Summary.** Roughly 1,000 previously unused solar eclipse timings measured between A.D. 1672 and 1806 are analysed to investigate the variation of  $\Delta T$  ( $=ET - UT$ ) over this interval. Comparisons are made between the results of the present analysis and those obtained using other data by Brouwer (1952) – as extended by Martin (1969) – Stephenson and Morrison (1984), and Goldstein (1985). It is shown that over this period of 134 years, eclipse observations closely support the solutions of Brouwer-Martin and Stephenson-Morrison, which are based on contemporary measurements (occultations of stars by the Moon). However, the eclipse data are in considerable discord with the solution of Goldstein, which is derived from relatively recent determinations of  $\Delta T$  only – since A.D. 1956. In our opinion, Goldstein's analysis inadequately represents the observed changes in  $\Delta T$  before about A.D. 1800 due to the oversimplified nature of his basic hypotheses.

**Key words:** decade fluctuations – Earth's rotation – Io acceleration – solar eclipses

## 1. Introduction

In 1975, Goldstein analysed approximately 50 eclipse observations of the Jovian satellite Io made by Picard and Roemer in the interval A.D. 1668–1678 and he derived appropriate values of the mean longitude and mean motion for the Picard-Roemer data. Goldstein also analysed approximately 150 Io eclipse observations made by Innes in the interval 1909–1927 and he derived a second set of mean longitude and mean motion values for the Innes data. Goldstein employed UT (Universal Time) in his analysis of both data sets.

In his 1975 paper, Goldstein then employed these two sets of longitude and mean motion values in an attempt to estimate the secular changes in the mean motion of Io between the Picard-Roemer epoch in 1674 and the Innes epoch in 1916. He used Brouwer's (1952: Table VIII) values for  $\Delta T$ , the difference between ET (Ephemeris Time) and UT. Since Goldstein's values for the secular acceleration  $\dot{n}(\text{Io})$  derived from the longitude comparison did not agree with that derived from his mean motion comparison, Goldstein published a bound on the possible secular acceleration of  $\dot{n}/n < 11 \cdot 10^{-11}$  per year.

Subsequently, Goldstein (1985) developed his own  $\Delta T$  function by employing published annual values of  $\Delta T$  since the begin-

ning of the atomic clock era (1956). Utilizing the 29 annual values of  $\Delta T$  between 1956 and 1984 as data, Goldstein hypothesized that the ET could be derived using Newcomb's (1895) ephemeris of the Sun by assuming that Newcomb's theory was exact except for a time error of the form  $\Delta T = a(t - s)^2$ . Goldstein further hypothesized that the reference epoch  $s$  had to be in the interval of Newcomb's scientific activity. In his 1985 paper, Goldstein determined values of  $a$  and  $s$ , leading to the formula

$$\Delta T = 45.39(t - 18.751)^2, \quad (1)$$

where time  $t$  is measured in Julian centuries. Goldstein then compared his simplistic formulation (which, for example, ignores decade fluctuations in  $\Delta T$ ) with the results of Brouwer (1952) over the interval 1820–1950. Brouwer's results were based on a large number of telescopic timings (of occultations of stars by the Moon) measured throughout that interval. Goldstein found a tolerable maximum difference (of the order of 10 s) between his own formulation for  $\Delta T$  and the values obtained by Brouwer as far back as 1820. However, the discord between Eq. (1) and the values of  $\Delta T$  derived from earlier telescopic timings by Brouwer (1952) – as extended by Martin (1969) – and Stephenson and Morrison (1984) is disturbingly large. In subsequent discussion, reference to the works of Brouwer (1952) and Martin (1969) will be (jointly) abbreviated to BM: the Stephenson and Morrison (1984) paper will be denoted as SM.

Then in 1986, Goldstein and Jacobs adopted Goldstein's (1985)  $\Delta T$  function and employed the *same* values for the longitude and mean motion of Io that Goldstein had derived in his 1975 paper. They re-derived the secular acceleration of Io and found  $\dot{n}(\text{Io})/n = +46 \cdot 10^{-11}$  per year, a dramatic difference from Goldstein's earlier result.

Since Goldstein's  $\Delta T$  value at the Picard-Roemer epoch (1674) differs widely – by some 200 sec – from those of BM and SM, it is of interest to utilize an existing early data set (*viz.* solar eclipses) which was not employed by either BM or SM in deriving their  $\Delta T$  results. One can then assess whether or not this independent data set supports Goldstein's value of  $\Delta T$  or those of BM and SM.

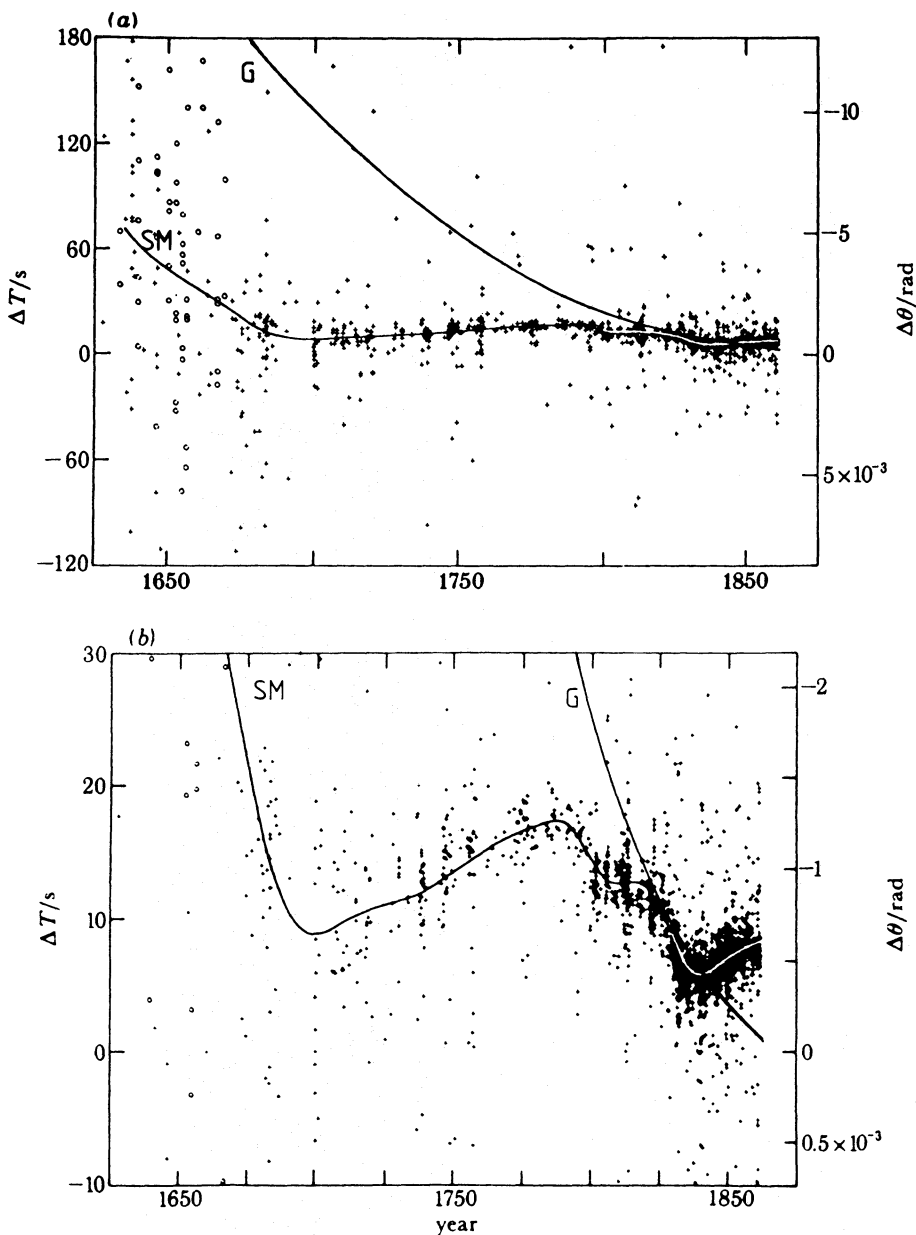
## 2. Determination of $\Delta T$ from occultations between A.D. 1620 and 1860

The most recent detailed analysis of changes in the Earth's rate of rotation over the telescopic period is that due to SM. This formed part of a long-term investigation covering the past 2700 yr. The

Send offprint requests to: J.H. Lieske

analysis of telescopic observations by SM, which was a refinement and extension of the solution of BM, covered the period since A.D. 1620. Occultations of stars at the dark limb of the Moon formed the only data set between A.D. 1670 and the start of the atomic clock era – i.e. 1956. These observations were regarded by SM as the most accurate and continuous series of suitable measurements over the entire interval from A.D. 1670 to 1956. A dark limb occultation has the attraction that it is a well defined and virtually instantaneous event (occurring in a small fraction of a second). The telescopic data analysed by SM were grouped into two sets – before and after 1860. Since systematic alternative measurements (i.e. solar eclipse contacts) are only readily accessible during the earlier period, we shall not be concerned here with the analysis by SM of observations after 1860 (which largely incorporated the investigation of Morrison, 1979a).

For the period between 1670 and 1860, SM analysed about 2,000 dark limb occultations. These data had been compiled by Morrison et al. (1981). The observations were made at more than 60 stations throughout Western Europe. Hence there seems little possibility of significant observer bias in the  $\Delta T$  results obtained by SM. For the previous half century only (actually from A.D. 1621 to 1668), approximately 70 solar eclipse timings were also included by SM in their analysis owing to the scarcity of useful occultation data at this early period. The optical definition of eclipse contacts is in general much inferior to that of dark limb occultations, except in the rather rare event of a total or annular eclipse. However, during much of the 17th century, methods of timing – largely by measuring altitudes of the Sun, Moon or clock stars – were very inaccurate. Hence SM felt that it was practicable to supplement the very earliest occultation data by eclipse observations.



**Fig. 1.** **a** Values of  $\Delta T$  between A.D. 1620 and 1860 as derived from individual timings of occultations (crosses) and last contacts for solar eclipses (open circles). The curves labelled SM and G are due to Stephenson and Morrison (1984) and Goldstein (1985) respectively [After Stephenson and Morrison, 1984]. **b** As in **a** but on an enlarged scale to show fluctuations after about A.D. 1800 in more detail

Determinations of the Earth's rotational clock error  $\Delta T$  ( $= ET - UT$ ) were made by SM for each occultation and eclipse observation on the assumption of a lunar acceleration  $\dot{n}$  of  $-26$  arcsec/century<sup>2</sup>. This value was derived by Morrison and Ward (1975) from observations of transits of Mercury since A.D. 1706. The individual values of  $\Delta T$  between A.D. 1620 and 1860 obtained by SM are plotted in Figs. 1a and 1b, which are taken from Figs. 2a and 2b of their paper. In these diagrams, crosses represent results from occultations and open circles those from solar eclipses. Figures 1a and 1b also show the mean  $\Delta T$  curves (labelled SM), obtained by smoothing individual  $\Delta T$  values by cubic splines (with 13 knots). The mean standard errors in  $\Delta T$  estimated by SM over various date ranges are as follows: 1 min between A.D. 1620 and 1669; 15 s from 1670 to 1699; 5 s from 1700 to 1759; 2 s from 1760 to 1819; and 1.5 s between A.D. 1820 and 1860.

The value of  $\dot{n}$  used by SM is in good accord with that derived from lunar laser ranging:  $\dot{n} = -25.3 \pm 1.2$  arcsec/century<sup>2</sup> (Dickey et al., 1984). In order to incorporate this latter figure, it would be necessary to amend the  $\Delta T$  results of SM by  $-0.73 \times (t - 19.555)^2$  sec, where  $t$  is measured in Julian centuries. These corrections are small compared with the above standard errors at the corresponding dates: they range from  $-8.2$  s at A.D. 1620 to  $-0.7$  s at 1860.

Figures 1a and 1b show that as far back as 1820, the agreement between Eq. (1) as deduced by Goldstein - the curves labelled  $G$  - and the results derived from occultations by SM is fairly satisfactory. However, before that data the discord rapidly increases. In particular, the data of SM reveal considerable fine structure around A.D. 1800. This is quite characteristic of the

decade fluctuations in the Earth's rotation observed in detail in more recent times (e.g., as mapped by Morrison, 1979a).

### 3. Solar eclipse timings: A.D. 1672-1806

In order to attempt to resolve these difficulties, we have investigated an independent and hitherto unused data set: solar eclipse timings between 1672 and 1806. This precise date range was chosen because: (i) earlier eclipses have already been analysed by SM; and (ii) systematic eclipse data are difficult to obtain for several decades after 1806. Because of inferior optical definition, there can be little doubt that most of the eclipse measurements are of poorer quality than the occultation timings investigated by SM (There are only a few timings of central total and annular eclipses in our data set). However, the deviation between the results of  $G$  and SM is so large during most of the selected period that the use of such material is viable. In all, about eclipse 1,000 timings measured at more than 100 stations have been analysed. Hence, as for occultations, the effects of observer bias should be negligible.

All of the eclipse data used in the present investigation were taken from the extensive catalogue of telescope timings of both occultations and eclipses produced by Morrison et al. (1981). In preparing that paper, solar eclipse observations between A.D. 1621 and 1806 were extracted from a large number of published works. For 17th century material, the detailed compilation by Pingre (1783) proved to be a particularly valuable source. Pingre took his material from a wide variety of observatory annals and other papers, some of which are rather inaccessible today. So

DISTRIBUTION OF DELTA-T OBSERVATIONS

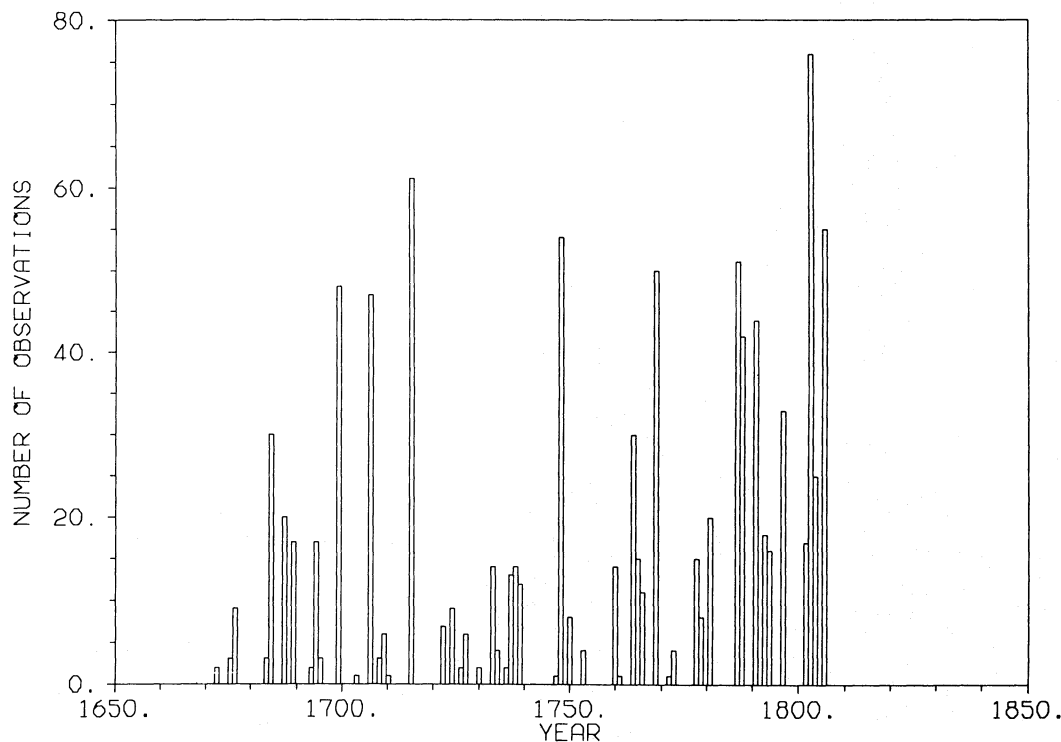


Fig. 2. Number of observations (all contacts) analysed for each solar eclipse between A.D. 1672 and 1806

thorough was his search of 17th century literature that Morrison et al. added very little material from this period to his findings. During the 18th century, preoccupation with the accurate determination of geographic longitude led to widespread measurement of solar (as well as lunar) eclipse times. These results were regularly published in the leading journals of the time, and hence are readily accessible today. However, soon after 1800, interest in the application of eclipses to this problem waned. Over the next few decades, relatively few eclipse timings were published. This is why the catalogue of solar eclipses observations by Morrison et al. (1981) terminated at A.D. 1806.

All of the eclipse timings listed by Morrison et al. were measured by European observers. During the period in question, few accurate observations were made in other parts of the world. In the original sources, timings are usually expressed to the nearest second. However, as for the occultations analysed by SM, the real accuracy achieved was probably much poorer than this – especially in the earlier decades. Most measurements are expressed in terms of apparent solar time, although local mean time was used for some of the later observations. Morrison et al. converted the various measurements from local apparent or local mean time to UT and we have made direct use of their reductions (For every observation, these authors also provided the geographic co-ordinates of the appropriate station).

Figure 2 shows the number of independent timings for each eclipse during the selected period. Although after about A.D. 1700 the long-term trend in frequency is very gradual, individual numbers of observations vary markedly (from only one or two to more than 70). These variations are due partly to the effects

of weather, but also eclipses of large magnitude tended to attract wider interest than those in which only a small part of the Sun was obscured.

#### 4. Analysis of the observations

Our analysis of the solar eclipse timings between A.D. 1672 and 1806 was made using the lunar ephemeris  $j = 2$  (IAU, 1968). However, the following small correction was applied to the lunar mean longitude ( $L$ ) in order to fully incorporate the selected value for  $n$  of  $-26$  arcsec/century<sup>2</sup> (Morrison, 1979b):

$$\Delta L = -1.54'' + 2.33''T - 1.78''T^2 \quad (2)$$

In the above equation,  $T$  is measured in Julian centuries from the epoch 1900.0. The ET for each of the roughly 1000 eclipse contacts was first computed and this was then compared with the observed time, as reduced to UT by Morrison et al. (1981). Hence a direct result for  $\Delta T$  was obtained for each observation. No allowance was made for lunar limb profile since errors from this source should be small (typically two or three seconds) and fairly random. Apart from occasional copying errors, the main sources of uncertainty are expected to occur in the detection of the contacts and – especially in earlier periods – in the measurement of local time.

Almost all solar eclipse observations analysed here were of first or last contact of the lunar limb with that of the Sun: very few timings related to the onset and end of totality or annularity which are, of course, rather rare events. Advance predictions of

DELTA-T VALUES FROM SOLAR ECLIPSES

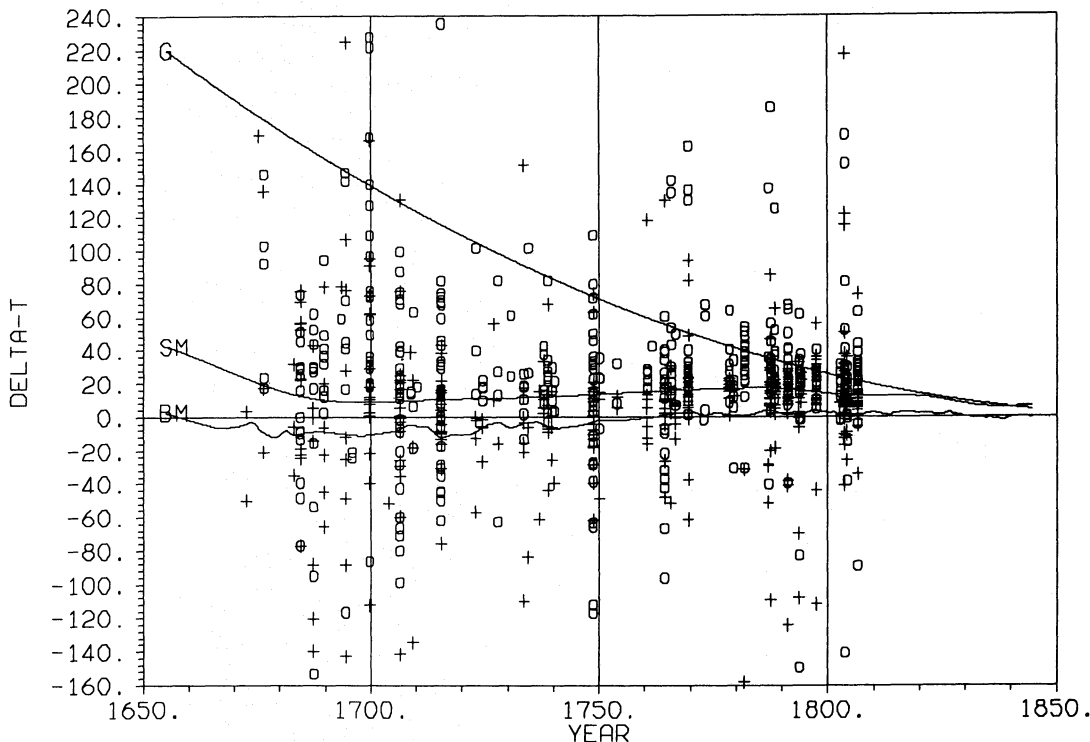


Fig. 3.  $\Delta T$  values derived from solar eclipses in the period A.D. 1672 to 1806. Crosses represent first contact observations and open circles other observations (mainly last contact). The curves labelled SM, BM and G are respectively due to Stephenson and Morrison (1984), Brouwer (1952) – as extended by Martin (1969) – and Goldstein (1985)

first contact were made by the various observers but these were often rather poor, occasionally leading to inadequate anticipation of the beginning of an eclipse. Hence timing errors would seem likely to be appreciably greater at first contact than for other eclipse phases. For that reason, we have grouped the measurements analysed into two sets: (i) first contacts only; (ii) all other contacts.

The results of the above analysis are shown in Fig. 3. That diagram is a plot of the individual values for  $\Delta T$  which we have derived from the solar eclipse timings measured during the period 1672 to 1806. About five per cent of all these values lie outside the range of the diagram ( $-160 \text{ s} \leq \Delta T \leq +240 \text{ s}$ ); many of the discordant results probably arose from copying errors. In Fig. 3, first contact observations are denoted by crosses and other contacts (mainly last) by open circles. It is clear from the diagram that contact bias is less significant than errors in measuring local time. The three curves labelled SM, BM and G which are superimposed on Fig. 3 are respectively due to Stephenson and Morrison (1984), Brouwer (1952) – as extended by Martin (1969) – and Goldstein (1985). SM is as shown in Figs. 1a and 1b. BM is based on a value for  $\dot{n}$  of  $-22.44 \text{ arcsec/century}^2$ . If the curve BM were to be corrected to our adopted value for  $\dot{n}$  of  $-26 \text{ arcsec/century}^2$  (as used also by SM), discrepancies in  $\Delta T$  between the two solutions BM and SM would be reduced to less than 6 s over the date range of the diagram. Figure 4 is an edited version of Fig. 3 in which results which lie more than 3 standard deviations from the mean for the appropriate eclipse date are rejected.

It is evident from Figs. 3 and 4 that our new results based on independent data (i.e. solar eclipse timings) are in excellent agreement with both BM and SM. However, there is considerable

deviation relative to G, especially at earlier epochs. In Figs. 5 and 6 we have extended the time-interval to cover the period from A.D. 1600 to the present, Fig. 6 being an edited version of Fig. 5 (cut-off at 3 standard deviations). These two diagrams, while illustrating the reasonable success of G in representing  $\Delta T$  (to within about 10 s) back to about A.D. 1800, demonstrate the failure of this parabola to accord with observations before this time.

### 5. Conclusion

In the determination of  $\Delta T$  in the period from A.D. 1672 to 1806, solar eclipse observations show a wider scatter than occultation measurements. However, they provide valuable independent verification of the results obtained from analysis of occultations in this period – by Brouwer (1952) and Martin (1969), and more recently by Stephenson and Morrison (1984). On the contrary, it is evident that the analysis of Goldstein (1985) inadequately represents the changes in  $\Delta T$  which can be mapped in fair detail at this period. In our opinion, this is in part due to Goldstein's neglect of the scale of the decade fluctuations in the Earth's rotation (which are probably largely due to electromagnetic coupling between the core and mantle). However, a major source of error is the oversimplified nature of his basic hypothesis – that long-term extrapolation of  $\Delta T$  based on less than three decades of data is possible. In consequence, Goldstein and Jacobs's (1986) application of Eq. (1) to observations of the Jovian satellite Io nearly three centuries before the time interval 1956–1984 should be viewed with caution.

DELTA-T VALUES FROM SOLAR ECLIPSES

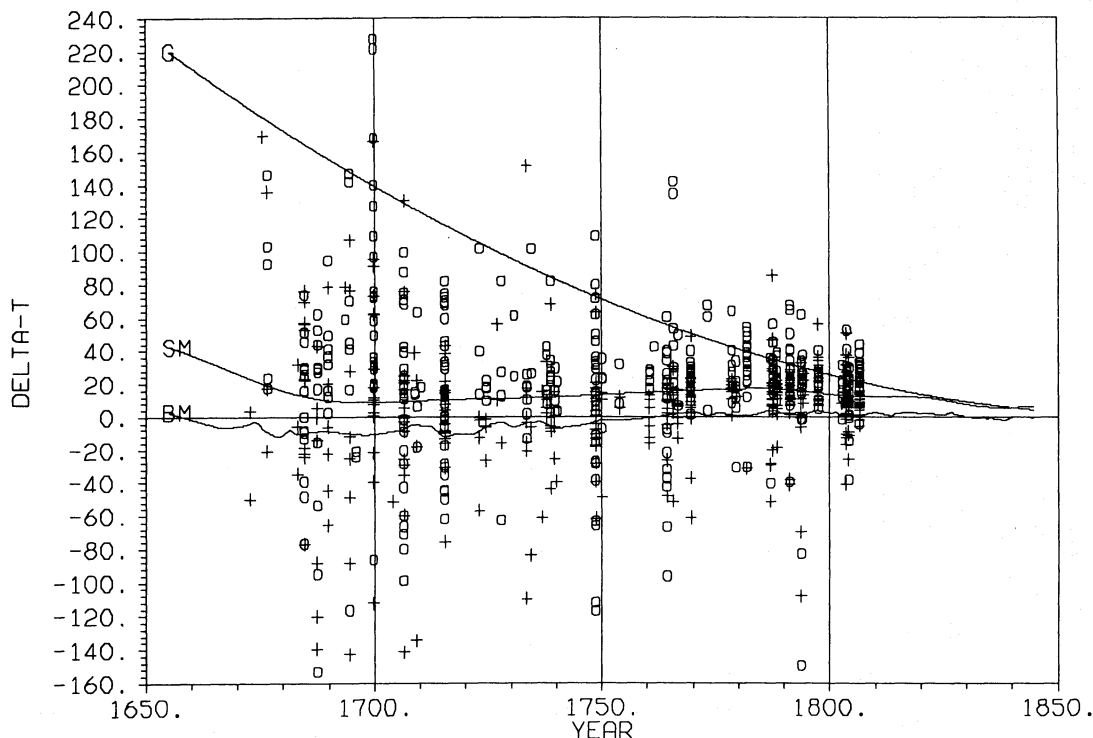


Fig. 4. As Fig. 3 but edited (i.e. with rejection of results which lie more than 3 standard deviations from the mean for the appropriate eclipse)



## DELTA-T VALUES FROM SOLAR ECLIPSES

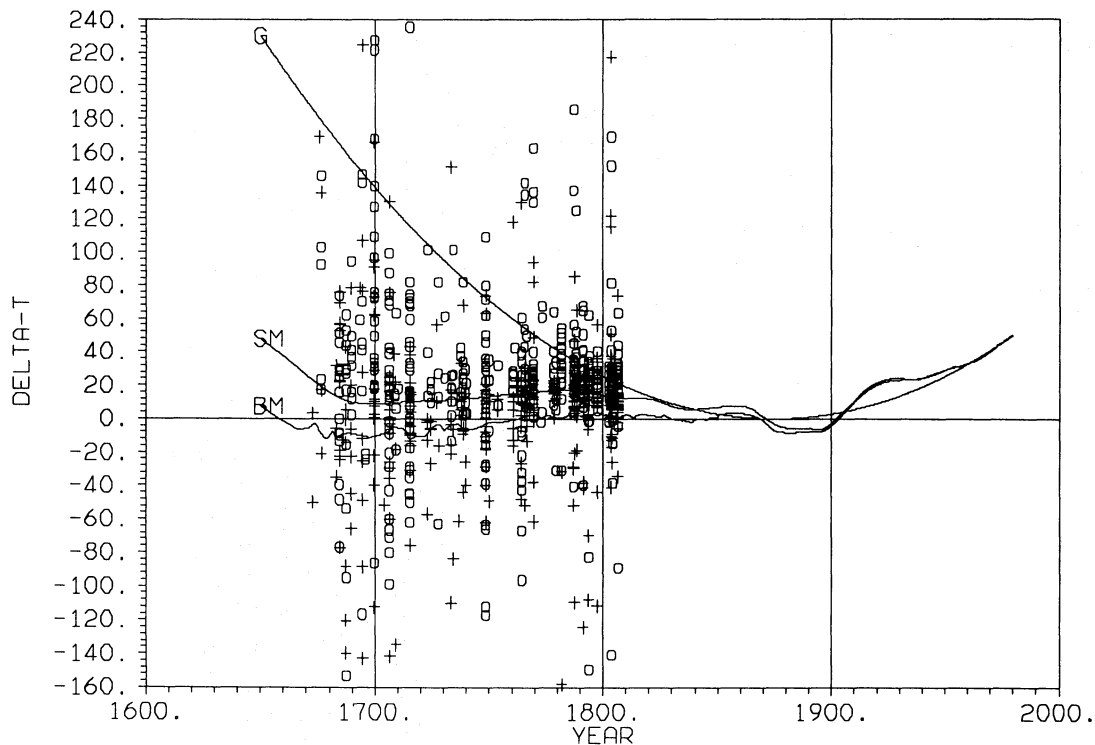


Fig. 5.  $\Delta T$  values derived from solar eclipses in the period A.D. 1672 to 1806. The time-interval is expanded relative to Fig. 3 to cover the period from A.D. 1600 to the present

## DELTA-T VALUES FROM SOLAR ECLIPSES

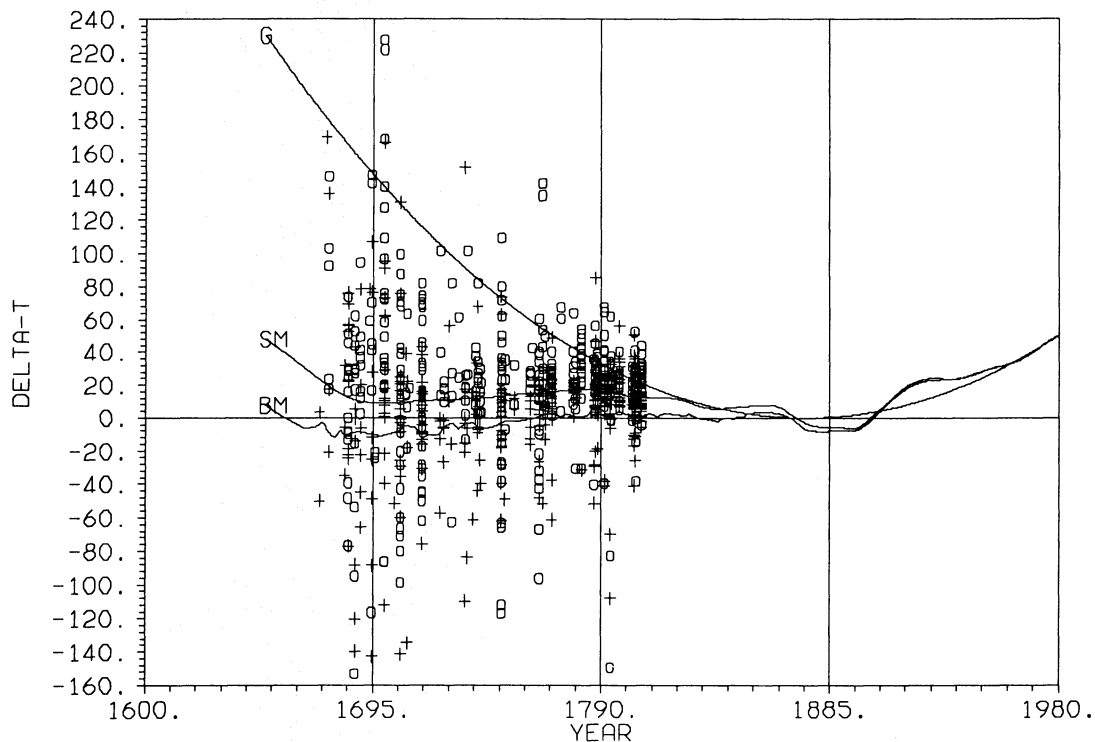


Fig. 6. As Fig. 5 but edited (i.e. with rejection of results which lie more than 3 standard deviations from the mean for the appropriate eclipse)

*Acknowledgements.* This paper represents the results of one phase of research conducted at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

#### References

- Brouwer, D.: 1952, *Astron. J.* **57**, 125  
Dickey, J.O., Williams, J.G., Newhall, X.X., Yoder, C.F.: 1984, *Proc. IAG/IUGG, Hamburg*, Ohio State Univ., Columbus, II, pp. 509–521  
Goldstein, S.J.: 1975, *Astron. J.* **80**, 532  
Goldstein, S.J.: 1985, *Astron. J.* **90**, 1900  
Goldstein, S.J. and Jacobs, K.C.: 1986, *Astron. J.* **92**, 199  
IAU: 1968, *Trans. Int. Astr. Un.* **13B**, 48  
Martin C.F.: 1969, Ph.D. Diss., Yale Univ.  
Morrison, L.V.: 1979a, *Geophys. J. Roy. Astron. Soc.* **58**, 349.  
Morrison, L.V.: 1979b, *Monthly Notices Roy. Astron. Soc.* **187**, 41.  
Morrison, L.V., Lukac, M.R., Stephenson, F.R.: 1981, *Roy. Greenwich Obs. Bull.* **186**.  
Morrison, L.V., Ward, C.G.: 1975, *Monthly Notices Roy. Astron. Soc.* **173**, 183  
Newcomb, S.: 1895, *Astron. Pap. Amer. Eph.* **6**, part I.  
Pingre, A.G.: 1783, *Annales Célestes du Dix-septieme Siècle* (Ed. M. Bigourdan, Paris, 1901)  
Stephenson, F.R., Morrison, L.V.: 1984. *Phil. Trans. Roy. Soc. London, A*, **313**, 47