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The observational basis for JPL's DE 200, the planetary ephemerides of the Astronomical Almanac

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Abstract. The Jet Propulsion Laboratory's ephemerides, DE 200/LE 200, now form the basis for the planetary and lunar positions listed in most of the major national almanacs throughout the world. They have resulted from a least squares' adjustment to a wide variety of observational data totaling over 50 000 positional measurements. These data types include optical data (transit and photographic), radar-ranging, spacecraft ranging and lunar laser-ranging.

This paper documents the planetary observational data used in the series of ephemerides produced at JPL over the six years preceding the creation of DE 118/LE 62, the set which was transformed directly into the JD 2000-based set, DE 200/LE 200. The major planetary ephemerides in this period of evolution were DE 96, DE 102, DE 108, DE 111 and DE 118. The paper also presents details of the procedures involved in the data reductions. For the optical data reductions, the paper includes corrections for planetary limb biases, phase effects, day-night differences, catalogue drifts and catalogue offsets. For the radar ranges, the basic light-time equation includes Earth orientation, station location and reflection point location; it is augmented by corrections for the time delays due to general relativity, the electron content of the solar corona and the troposphere. Further, techniques to overcome the uncertainties due to planetary topography are described, including closure point analysis and occultation point comparisons. For the spacecraft data, the basic reductions, similar to those for radar, are augmented by formulations for locating the transponder, whether in orbit or landed on the surface of a planet.

The total set of observational data contained at JPL represents a unique collection. Some of the data are published in the various astronomical journals and have been retrieved directly from these sources; some were sent personally to JPL by the observer; some of the data originated at JPL itself. Some of the data are in the possession of many astronomers throughout the world; some are kept only at various locations within JPL; some may be only in the possession of the planetary ephemeris group at JPL. The complete data set has been archieved onto magnetic tape. The providers of the data are acknowledged.

Key words: ephemerides – solar system – astronomical constants

1. Introduction

The Jet Propulsion Laboratory's planetary and lunar ephemerides have now been sent throughout the world to various scientists and institutions; they are being used for a wide variety of different applications. One set of ephemerides in particular, DE 200/LE 200 (Development Ephemeris 200/Lunar Ephemeris 200), has been adopted by most national almanac offices to form the basis of their national ephemerides. DE 200/LE 200 is used directly in the Astronomical Almanac of the United States and Great Britain (Astronomical Almanac, 1984); it was fit by other numerical integrations to form the Japanese Ephemeris (JHD, 1984); it was fit by the Bureau des Longitudes' analytical planetary formulation, VSOP 82 (Bretagnon, 1982), and lunar formulation, ELP 2000 (Chapront-Touzé and Chapront, 1983), for use in the Connaissance des Temps.

Occasionally, it is useful to know the differences between the various ephemerides since they are often indicative of the realistic accuracies of the ephemerides. For the most part, the quality (i.e., accuracy) of the ephemerides is a direct result of the observational data to which they were fit.

This paper documents the planetary portion of the observational data used in the fitting process of the JPL ephemerides, DE 96/LE 44, created in November 1975; DE 102/LE 51, September 1977; DE 108/LE 44, August 1978; DE 111/LE 55, May 1980; and DE 118/LE 62, September 1981. The process of rotating DE 118/LE 62 into its J 2000-based version, DE 200/LE 200, is described by Standish (1982).

The basic fundamentals of the ephemeris creation process have been presented elsewhere (see, especially, Newhall et al., 1983); we now assume the validity and completeness of the equations of motion (i.e., the gravitational theory and inclusion of all relevant masses); we have tested the accuracy of the numerical integration program. Consequently, these problems are no longer of concern.

Therefore, the crucial part of a modern-day ephemeris development process is the fitting of the ephemerides to the observational data. It is the quality, quantity, variety and coverage in time of the observational data, along with the degree of refinement of the reduction processes, which directly determine the accuracy of the resultant ephemerides.

Since the creation of DE 118/LE 62, JPL has acquired additional observational data, including a number of newer data types. These, coupled with refinements to the reduction processes, have resulted in improvements to the positions and motions given by DE 118 (and, therefore, by DE 200); these will be described in a later paper. The present paper serves as a foundation for the more recent developments.

There are four major types of observational data included in the adjustments of the ephemerides up to and including

DE 118/LE 62 (and DE 200/LE 200); optical measurements, radar-ranging, spacecraft-ranging and lunar laser-ranging. The three planetary data types are discussed in this paper, along with details of the reduction processes. The optical data is presented in Sect. 2; the light-time equation is given in Sect. 3; the radar and spacecraft data are presented in Sects. 4 and 5, respectively. Section 6 presents statistics of the observations, including details of the solution parameters, accuracies of the observational data and more recent types of data. Finally, the providers of the planetary data sources are mentioned with the acknowledgements.

2. Optical data

There were three types of optical data used in the JPL ephemerides up to and including DE 118. These are the transit observations from the U.S. Naval Observatory (USNO), some astrometric plate data of Saturn and a set of normal points for Neptune and Pluto, derived from optical measurements, which were provided by the USNO.

2.1. *Transit circle data*

The only transit data used in the JPL ephemerides up to and including DE 118 were the observations from the U.S. Naval Observatory (USNO), taken with the six-inch and nine-inch meridian circles. Furthermore, only those since 1911 have been used in the JPL ephemerides; the date signifies the installation of the impersonal micrometer onto the meridian circle instruments.

The observations are recorded in the Publications of the USNO, second series. The data listed in these publications have been reduced to the Washington Catalogue of the concurrent epoch. Included in each publication is a set of tables with which one may relate that Washington Catalogue to the standard general catalogue of the epoch (i.e., Boss GC, FK 3, FK 4). Subsequent tables provide the transformation from these catalogues to the FK 4; references to these publications are included in Table 1 of this paper.

Previous to the creation of DE 96, all of the transit observations were transformed onto the reference system of the FK 4 for use in the JPL ephemerides. The transformation tables given by the USNO were used for the creation of DE 96 and DE 102. Starting with DE 108, the tables for W(25) through W 4(50) were replaced by the more modern formulae of Schwan (1977), though a brief comparison reveals that the differences between the two transformations rarely exceeds $0\overset{''}{.}1$.

The planetary observations which were concurrent with the W 5(50) Catalogue were originally published in a series of USNO Circulars (No. 105, 108, 115, 118, 124, 127, and 136). Though preliminary in nature, these data were used as published in the creation of DE 96 through DE 111, since the observations were made directly with respect to the FK 4 system. For DE 118, the planetary observations listed in those Circulars were replaced by their final values which appeared along with the W 5(50) Stellar Catalogue.

Data since the end of the W 5(50) in 1971 have been provided in machine-readable format by the USNO. These, especially, must be considered provisionary, since the transit circle was refurbished during the interval 1972–1974.

2.1.1. Basic reduction of the transit observations

Transit observations are differential in nature, the planetary observations undergoing the same processing as those of the observed stars, both being related to the standard catalogue of the epoch during which the observations were made. The observations are published as geocentric, apparent right ascensions and declinations, taken at the time of meridian passage. For comparison, then, a computed position from the ephemerides is obtained by iterating to find the time at which the local apparent hour angle of the planet is zero. The computed position is then given by the ephemerides at that transit time. The formulation for computing apparent places has been essentially identical to that described in the Astronomical Almanac (1984, p. B36).

Table 1. Transit circle observations from the U.S. Naval Observatory that have been used in the JPL ephemerides, DE 96 through DE 118. The vol./part refers to the second series of the USNO publications

Wash cat.	Time span	Tele-scope	Vol./ part	Number of observations				
				DE 96	DE 102	DE 108	DE 111	DE 118
W(10)	1911–18	6″	XI	2436	2436	2436	2436	2436
W(20)	1913–25	9″	XIII	3381	3381	3381	3381	3381
W(25)	1925–33	6″	XVI/I	6911	6911	6911	6911	6911
W(40)	1935–44	9″	XV/V	4547	4547	4547	4547	4547
W(50)	1933–41	6″	XVI/I	3777	3777	3777	3777	3777
W2(50)	1941–49	6″	XVI/III	3444	3444	3444	3444	3444
W3(50)	1949–56	6″	XIX/I	3678	3678	3678	3678	3678
W4(50)	1956–62	6″	XIX/II	4051	4051	4051	4051	4051
[Cir.]	1963–71	6″		5628	5628	5628	5628	0
W5(50)	1963–71	6″	XXIII/III	0	0	0	0	5811
[Cir.]	1975–77	6″		0	0	1089	1543	1543
Total				37853	37853	38942	39396	39579

2.1.2. Limb corrections applied by the USNO

In the eyepiece of the transit instruments one sees a square box formed by the intersections of two pairs of parallel cross-hairs. It is at the center of this square where one attempts to locate the feature of the object being measured. The feature is centered horizontally within the box for the right ascension measurement; vertically for the declination.

Almost always for Mercury and sometimes for Venus, the center of light of the visible disk is measured in right ascension and in declination. Occasionally for Mercury and usually for Venus, only the illuminated limb of the visible disk is measured in right ascension and in declination. For Sun, Mars, Jupiter and Saturn, both the illuminated limb and the terminator are always measured, both in right ascension and declination. Uranus and Neptune are treated like stars; i.e., the center of light is measured.

For each measured feature, a correction is applied in order to relate the position of that feature to the center of mass. The correction is a function of two angles, Θ and i, where Θ is the position angle of the midpoint of the illuminated edge and where the phase angle, i, is the angle between Sun and Earth, subtended at the observed body.

The angles, Θ and i, are defined on pages 311–312 of the Explanatory Supplement to the Ephemeris (1961). Care must be taken to distinguish between the presently discussed angle, Θ, and the previously used θ, the position angle of the line of cusps ($\theta=\Theta+90°$). Indeed, the USNO tables for these corrections are actually labelled using this latter quantity.

The form of the corrections depends upon the type of observation and upon which planet is being observed. The corrections are applied immediately to the observations by the USNO. Properly, the following formulae should include the factor of $\cos\delta$ applied to each formula for $\Delta\alpha$. However, this was never stated explicitly in the references and therefore was not assumed here.

(i) Center of light measurements. For center of light measurements, the tables are given in the Publications of the USNO, Vol. IV, Appendix II.

For Mercury, Table XXIV applies, computed from the empirical formula,

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=\begin{vmatrix}-s/r\,\sin\Theta(1-\cos i)(5+\cos i)/12\\ -s/r\,\cos\Theta(1-\cos i)(5+\cos i)/12\end{vmatrix}.$$

where s is the semi-diameter of the planet at unit distance and where r is the Earth–planet distance.

For Venus, Table XXIII applies, computed from the following (rigorously derived) formula which gives the offset from the center of the full planetary disk to the midpoint between tangents to the illuminated edge and terminator:

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=\begin{vmatrix}-\mathrm{sgn}(\sin\Theta)\;s/r[1-\sqrt{1-(\sin i\sin\Theta)^2}]/2\\ -\mathrm{sgn}(\cos\Theta)\;s/r[1-\sqrt{1-(\sin i\cos\Theta)^2}]/2\end{vmatrix}.$$

(ii) Illuminated limb measurements. For an observation of an illuminated limb, the geometric correction is simply,

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=\begin{vmatrix}\mathrm{sgn}(\sin\Theta)\;s/r\\ \mathrm{sgn}(\cos\Theta)\;s/r\end{vmatrix}.$$

(iii) 4-limb measurements. For observations of a gibbous disk where both the illuminated edge and the terminator are measured, Table XXV applies. Even though the table appears different, it is derived from the same geometric considerations as in the case of the center of light of Venus and produces the exact same correction. In this case, however, the table provides the factor, f, with which one multiplies the measured difference in the two limbs in order to obtain the observed semi-diameter. The factors are given by,

$$\begin{vmatrix}f_\alpha\\ f_\delta\end{vmatrix}=\begin{vmatrix}1/[1+\sqrt{1-(\sin i\sin\Theta)^2}]\\ 1/[1+\sqrt{1-(\sin i\cos\Theta)^2}]\end{vmatrix}.$$

2.1.3. Corrections to the transit data applied by JPL

In addition to the basic reduction, there are a number of further corrections which have been made to the transit data. Those are given next.

Limb biases (DE96). In the solution for DE96, there were four parameters which were intended to adjust the above mentioned limb corrections due to any unknown biases. The crude corrections were simply constants, one for each dimension, right ascension and declination, for both Mercury and Venus. These additive constants were applied in the direction of the illuminated edge when that was the feature which had been measured. Unfortunately, however, the file of observations contained only some of the proper data with respect to which limb had been measured. This has since been corrected, but, for DE96, these parameters are practically meaningless. In any case, they were abandoned in favor of the phase corrections, discussed in the next section, which were used subsequent to DE96.

Phase corrections (DE102–DE118). Even after the limb corrections of the preceding section had been applied, there remained a substantial systematic error which was correlated with the phase of the observed object (see e.g., Standish et al., 1976). It is quite evident that these trends arise from the process of measuring the position of a disk which is not illuminated uniformly.

Starting with DE102, empirical formulae have been used to fit and remove the "phase effects" from the transit circle observations. It seems that these result mainly from "irradiation effects" whereby an observer tends to measure a bright illuminated limb differently from a darker terminator.

(i) Center-of-light measurements. For the center of light measurements, for both Mercury and Venus, the USNO correction was removed from the observations, and, instead, the following formula was applied:

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=s/r\begin{vmatrix}\sin\Theta\\ \cos\Theta\end{vmatrix}[C_0+C_1I+C_2I^2+C_3I^3],$$

where I is the phase angle expressed in units of 90° ($I=i/90°$). The coefficients, C_i, are solved for in the ephemeris solutions. The resulting cubic polynomial seems to agree quite well with that found by Lindegren (1977). The values of the coefficients are given in Table 13.

(ii) Illuminated limb measurements. For an illuminated limb measurement, for both Mercury and Venus, the USNO correction was retained and the following formula was applied in

addition:

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=\begin{vmatrix}\sin\Theta\\ \cos\Theta\end{vmatrix}[L_0+L_1I+L_2I^2+L_3I^3].$$

Note that the factor of s/r is not present in this case since this formula was judged to give a slightly better fit without the factor. In any case, the effect of distance can be compensated by adjustment of the coefficients, since the angle I is correlated to the distance of the planet. The coefficients, expressed in seconds of arc, are given in Table 13.

(iii) 4-limb measurements. Similarly, for measurements of four limbs, the USNO correction was retained and the following formula was applied in addition:

$$\begin{vmatrix}\Delta\alpha\\ \Delta\delta\end{vmatrix}=\begin{vmatrix}\sin\Theta\\ \cos\Theta\end{vmatrix}B_k\sin 2i$$

where $k=4, \ldots, 8$ for Mars, . . . , Neptune. The values of the coefficients are given in Table 13. This empirical formula may be compared with that of Chollet (1984), the form of which has been derived from actual physical considerations.

Again, the forms of these preceding "phase correction" formulae were all chosen strictly from empirical considerations.

Day corrections. For each Washington catalogue, corrections are applied to the observations obtained during daylight hours; namely, observations of Sun, Mercury and Venus. Typically, these are given in tables as functions of the object's declination and of the time of day. Day corrections have not been applied, however, to the preliminary observations given in the Circulars. Therefore, for the data obtained from the Circulars, the following day corrections were solved for and removed from the observations of Sun, Mercury and Venus:

$$\Delta\alpha=A_1+A_2\sin\delta+A_3\cos h_\odot$$

$$\Delta\delta=D_1+D_2\sin\delta+D_3\cos h_\odot$$

where δ is the declination of the body and where $h_\odot$ is the solar hour angle (i.e., apparent time of day).

The coefficients in these formulae were solved for in the solutions for DE96 through DE111, being applied to all of the observations subsequent to W 4(50). Beginning with DE 118, A_3 and D_3 were not estimated, since they are highly correlated to A_1 and D_1. Also for DE 118, these formulae were applied to only the data subsequent to W 5(50). Table 13 contains their values.

Catalogue drift. It is well known that the mean motions of the inner planets are determined almost entirely by the strength of the ranging data in the least squares adjustments to the ephemerides (see e.g., Newhall et al., 1983, p. 162 or Williams and Standish, 1989). Therefore, any inconsistency between these mean motions and those implied by the optical data will appear as secular trends in the optical residuals. It is suspected that such drifts are due, among other things, to an incorrect value of precession and the presence of an equinox motion in the FK4 reference system. These have been accounted for in two ways.

Corrections to precession and equinox drift. In all of the ephemerides, the secular-like drifts in the optical residuals have been modelled by the standard formulae,

$$\Delta\alpha=(\Delta k-\Delta n\sin\alpha\tan\delta)\,T_{50}$$

$$\Delta\delta=(\Delta n\cos\alpha)\,T_{50}$$

where T_{50} is the time in centuries past 1950. These corrections are subtracted from the observed values of all of the transit observations of the planets and the Sun.

If one assumes that these parameters come exclusively from precession error and equinox drift, then the following relations apply:

$$\Delta k=-\dot{E}+\Delta p\cos\varepsilon-\Delta l \quad\text{and}\quad \Delta n=\Delta p\sin\varepsilon$$

where $\dot{E}$ is the equinox motion; Δp is the correction to luni-solar precession, Δl is the correction to planetary precession and ε is the value of the Earth's obliquity.

Both Δk and Δn were estimated in the solutions for DE96, DE102 and DE108. For DE111, the value of Δn was set to $+0\overset{''}{.}438\ \mathrm{cty}^{-1}$, corresponding to Fricke's (1971) determination of the correction to Newcomb's value of the luni-solar precession, $\Delta p=+1\overset{''}{.}10\ \mathrm{cty}^{-1}$. For DE118, Δk was also fixed, this to $-0\overset{''}{.}266\ \mathrm{cty}^{-1}$, corresponding to Fricke's (1982) determination of the equinox motion of the FK4, $\dot{E}=+1\overset{''}{.}275\ \mathrm{cty}^{-1}$. This value of Δk should have been $-0\overset{''}{.}237\ \mathrm{cty}^{-1}$, since the correction for planetary precession of $\Delta l=-0\overset{''}{.}029\ \mathrm{cty}^{-1}$ (see Lieske et al., 1977) was inadvertently omitted; such a difference, however, is quite negligible in this case.

With Δk and Δn constrained to the values determined by Fricke, the computed residuals of the transit data are nearly identical to those which would be found using the new J2000.0 (IAU, 1976) reference system.

Catalogue offsets (DE 118 only). With Δk fixed for the solution to DE118, there was the possibility of a remaining secular trend in the optical residuals. It was decided to introduce constant offsets a in both right ascension and in declination for each of the Washington catalogues. The values for these are included in Table 13.

2.2. *Astrometric plate data for Saturn*

During the years 1973–1979, the University of Virginia, under contract from JPL, provided measurements of photographic plates, taken of the planet Saturn and its satellites (Ianna, 1974, . . . , 1980). The measurements of the satellites and the background stars were provided to JPL and were reduced by the author, using the satellite ephemerides of Null (1978) and the Zodiacal Catalogue of the USNO. The "observed" positions of the satellites from the reductions were then compared to "computed" positions derived from the satellite and planetary ephemerides. Since the errors of the satellite ephemerides were expected to be small in comparison to those of the planetary ephemerides, and since the satellite errors enter in a quasi-random way when many observations are used, the differences (O-C's) are attributed to the errors in the planetary ephemerides.

For each opposition of Saturn, Table 2 lists the number of plates, the number of exposures, the number of different reference stars used, the number of pairs of observations of the reference stars, the number of pairs of observations of the satellites and the total number of observations.

Table 2. Satellite Astrometry Observations of Saturn from McCormick Observatory (used in DE 111 and DE 118 only). The total number of observations includes each pair (x and y) of star measurements, each pair of satellite measurements and two catalogue corrections for each reference star

Year	No. of plates	No. of exp.'s	No. of stars	No. of star obs.	No. of sat. obs.	Total obs.
1973–4	25	68	23	222	103	696
1974–5	16	62	21	387	97	1010
1976	25	78	30	329	147	1012
1977	12	44	20	149	100	538
1978	15	60	24	158	148	660
1979	13	48	21	201	215	874
Total	106	360	139	1446	810	4790

Table 3. Normal points of Neptune and Pluto, transmitted from the U.S. Naval Observatory (used in DE 118 only)

Neptune	1825–1974	137	Pairs of longitude and latitude
Pluto	1914–1974	56	Pairs
Total pairs: 193			Total observations: 386

2.3. *Normal points for Neptune and Pluto*

As mentioned before, the main optical data used by JPL are the transit observations from the USNO, beginning only in 1911. Clearly, for the outermost planets, this is an insufficient time span of data for the determination of a full orbit. Consequently, the USNO transmitted a set of normal points which gave the differences between one of their experimental ephemerides and an intermediary JPL ephemeris (DE 114). The optically-based USNO ephemeris gave what were, at that time, believed to be reasonable fits to the full time-spans of observations of Neptune and Pluto. These data were then used to adjust the solution for DE 118. The data were transmitted in the form of corrections to longitude and latitude, one pair of points every 400 days. The time span for Neptune was 1825–1974; for Pluto, 1914–1974. These data are summarized in Table 3.

3. Basic reduction of a range measurement

Ranging measurements are actual timings of the round-trip light-time of the electromagnetic signal from the time that it leaves the transmitter until the time that it arrives back at the receiver. The timing is done in the proper time, UTC, of the atomic clock at the observing station.

For an observation, received at the time t (expressed in TDB time units), the round-trip light-time is given by the difference, $\mathrm{UTC}(t)-\mathrm{UTC}(t-\tau_D-\tau_U)$, where

$$\tau_U=|\boldsymbol{r}_R(t-\tau_D)-\boldsymbol{r}_A(t-\tau_D-\tau_U)|/c$$
$$+\Delta\tau_U[\mathrm{rel}]+\Delta\tau_U[\mathrm{cor}]+\Delta\tau_U[\mathrm{tropo}],$$

and

$$\tau_D=|\boldsymbol{r}_A(t)-\boldsymbol{r}_R(t-\tau_D)|/c+\Delta\tau_D[\mathrm{rel}]+\Delta\tau_D[\mathrm{cor}]+\Delta\tau_D[\mathrm{tropo}],$$

τ_U and τ_D are the light-times (in TDB units) of the upleg and the downleg, respectively; $\boldsymbol{r}_A$ is the solar-system barycentric position of the antenna on the Earth's surface and $\boldsymbol{r}_R$ is solar-system barycentric position of that point on the planet's surface from which the signal is returned; c is the velocity of light; and the three $\Delta\tau$'s are the corrections to the light-times due to relativity, the electron content of the solar corona, and the Earth's troposphere, respectively. The two formulae are solved iteratively; first for τ_D, then for τ_U.

The location of the antenna is computed in a straightforward manner, using a planetary ephemeris and the proper formulae of precession, nutation, timing and polar motion with which one orients the Earth into the reference frame of the ephemerides.

The location of the point of reflection on the surface of a spherical planet is approximated by the intersection on the planet's surface of the line connecting the center of mass of the planet and the center of mass of the Earth, both computed at the instant of reflection, $t-\tau_D$. In the JPL ephemerides, the surfaces of Mercury and Venus have been modelled as spheres; for Mars, a tri-axial ellipsoid is used (see Standish, 1973).

The time-delay due to relativity, given by Shapiro (1964), is obtained by integrating along the signal path over the value of the potential. For each leg of the signal path, the delay is given by the formula,

$$\Delta\tau[\mathrm{rel}]=\frac{(1+\gamma)GM}{c^3}\ln\left|\frac{e+p+q}{e+p-q}\right|$$

where γ is the PPN parameter of general relativity and where e, p and q are the heliocentric distance of the Earth, the heliocentric distance of the planet and the geocentric distance of the planet, respectively, evaluated at $t-\tau_D$ for the planet, at $t-\tau_D-\tau_U$ for the Earth during the upleg and at t for the Earth during the downleg.

The delay from the solar corona (see Muhleman et al., 1977) is obtained by integrating along the signal path from point P_1 to point P_2 over the density of ionized electrons, $N_e[\mathrm{cm}^{-3}]$:

$$\Delta\tau[\mathrm{cor}] = \frac{40.3}{cf^2}\int_{P_1}^{P_2} N_e\,\mathrm{d}s,$$

where c is the speed of light $[\mathrm{cm\,s}^{-1}]$, f is the frequency [MHz], s the linear distance [cm] and where the density is given by

$$N_e = \frac{A}{r^6} + \frac{B}{r^2}$$

where r is the heliocentric distance, expressed in units of the solar radius. This expression was used for the radar data in all of the ephemerides. For the Mariner 9 range points, it was used before the solution for DE 111; for DE 111 and DE 118, the constant, B, was replaced by the (solar) latitude-dependent expression (Anderson, 1978),

$$ab/[\sqrt{a^2\sin^2\beta + b^2\cos^2\beta}],$$

where β is the solar latitude. The values for the constants, A, B, a and b are included in Table 13. Formulae for expressing the above integrals have been derived by Anderson.

The delay from the Earth's troposphere is discussed by Chao (1970). For each leg, it is

$$\Delta\tau[\mathrm{tropo}] = 7\ \mathrm{ns}/[\cos z + 0.0014/(0.045 + \cot z)],$$

where z is the zenith distance at the antenna.

4. Radar ranging data

Radar ranging data may be used directly in an ephemeris adjustment where the observation is essentially a distance measurement between the radar antenna and the point of reflection on the planet's surface. However, the accuracy of such an observation is usually degraded due to the planet's topography. Therefore, in addition to incorporating the radar observations directly into the ephemerides, JPL has also used two methods for alleviating the problem of topography: 1) the use of "radar closure" points, and 2) the comparison of the radar data with the occultation measurements by orbiting spacecraft.

4.1. Radar ranging measurements

The radar ranging measurements used in the JPL ephemerides have come from five different antennae. These are located in Arecibo, Puerto Rico; Tyngsboro, Mass. (Haystack); Westford, Mass. (Millstone) and Goldstone, Calif. (DSS 13 and DSS 14). In addition, some of the measurements of Goldstone were taken in the bi-static mode: transmitting with DSS 14 and receiving with DSS 13. Table 4 shows what measurements have been fit directly

Table 4. Radar ranging observations used directly in the JPL ephemerides

Planet	Time span	Antenna	Number of observations				
			DE 96	DE 102	DE 108	DE 111	DE 118
Mercury	1966–71	Arecibo	106	106	0	106	106
	1966–71	Haystack	217	217	217	217	217
	1971	Gold 13/14	9	9	9	9	9
	1972–74	Gold 14	22	22	29	30	30
	Mercury total		354	354	255	362	362
Venus	1966–70	Arecibo	248	248	248	248	248
	1966–71	Haystack	219	219	219	219	219
	1964–67	Millstone	101	101	101	101	101
	1964–70	Gold 13	294	294	294	294	294
	1970–71	Gold 13/14	14	14	14	14	14
	1971–73	Gold 14	44	44	44	44	44
	1973–77	Gold 14	0	0	24	25	25
	Venus total		920	920	944	945	945
Mars	1965	Arecibo	30	30	0	0	0
	1967–71	Haystack	2745	2745	0	0	0
	1969	Gold 13	4	4	0	0	0
	1969–71	Gold 13/14	300	300	0	0	0
	1971	Gold 14	699	699	0	0	0
	Mars total		3778	3778	0	0	0
All total			5052	5052	1199	1307	1307

Table 5. DSS 14 (Goldstone) radar observations of Mars, 1971–1978. For each day's track, the table shows the starting and ending times (UT) of reception, the mean latitude and the beginning and ending longitudes of the points on Mars from which the signal has been reflected

Date	Start	End	No.	Latitude	Longitude coverage	
1971 Jun. 20	08:58:53	14:41:14	520	−18°.21	279°.42	2°.79
1971 Jun. 23	08:16:18	14:45:55	466	−18.19	241.02	335.90
1971 Jun. 26	08:46:56	15:02:04	569	−18.13	220.54	311.90
1971 Jun. 29	07:31:23	14:02:15	532	−18.04	174.30	269.50
1971 Jul. 02	07:33:35	13:57:50	583	−17.93	147.09	240.69
1971 Jul. 05	09:39:28	14:00:52	360	−17.79	150.13	213.81
1971 Jul. 08	07:34:43	13:56:55	579	−17.62	92.22	185.34
1971 Jul. 11	07:44:06	14:02:58	574	−17.43	67.09	159.41
1971 Jul. 13	07:27:32	13:58:52	535	−17.29	44.85	140.21
1971 Jul. 17	11:32:48	14:07:44	234	−16.97	68.37	106.13
1971 Jul. 20	06:19:46	07:56:55	149	−16.74	325.01	348.69
1971 Jul. 23	07:58:29	13:24:02	487	−16.46	322.13	41.49
1971 Jul. 26	05:26:45	13:07:27	697	−16.19	258.29	10.61
1971 Jul. 30	07:38:35	13:04:04	495	−15.82	254.77	334.13
1971 Aug. 03	06:00:51	12:41:07	598	−15.46	195.41	293.01
1971 Aug. 07	07:09:26	12:13:14	453	−15.11	176.69	250.77
1971 Aug. 10	05:25:43	12:00:42	602	−14.87	124.85	221.17
1971 Aug. 13	05:30:12	11:49:26	355	−14.65	99.41	191.88
1971 Aug. 16	05:13:46	11:34:59	573	−14.47	68.85	161.81
1971 Aug. 19	05:07:17	11:18:41	375	−14.31	40.69	131.25
1971 Aug. 22	07:44:26	10:56:02	269	−14.20	52.37	99.09
1971 Aug. 25	06:24:19	10:44:11	384	−14.13	6.14	69.49
1971 Aug. 28	03:56:55	10:29:23	596	−14.11	303.41	39.09
1971 Sep. 01	03:29:26	10:09:10	606	−14.15	260.85	358.29
1971 Sep. 03	02:47:06	10:12:11	676	−14.20	232.53	341.02
1971 Sep. 06	02:39:27	09:59:19	667	−14.32	203.58	310.78
1971 Sep. 10	03:32:35	09:36:21	530	−14.54	180.22	268.86
1971 Sep. 13	01:44:54	09:06:51	668	−14.76	126.62	234.30
1971 Sep. 16	02:49:46	09:16:36	580	−15.02	114.94	209.19
1971 Sep. 20	01:58:39	08:55:44	569	−15.43	65.67	167.26
1971 Sep. 24	03:28:15	08:33:05	461	−15.90	50.46	124.71
1971 Sep. 27	01:22:22	07:52:38	569	−16.27	351.90	86.94
1971 Oct. 04	01:18:44	08:29:11	649	−17.26	285.51	30.31
1971 Oct. 07	00:41:37	08:12:28	293	−17.72	248.23	357.99
1971 Oct. 10	23:59:02	08:10:01	715	−18.34	200.08	319.60
1973 Jul. 12	15:59:07	18:40:17	191	−21.78	304.96	344.17
1973 Jul. 18	12:07:41	15:32:54	227	−21.05	190.40	240.32
1973 Jul. 24	10:45:26	16:10:19	386	−20.25	112.31	191.35
1973 Jul. 27	11:04:31	18:04:44	501	−19.83	87.99	190.23
1973 Jul. 30	10:34:42	14:11:03	297	−19.42	51.83	104.47
1973 Aug. 02	10:09:54	15:36:42	481	−19.00	16.95	96.47
1973 Aug. 06	10:06:28	16:44:15	498	−18.44	337.74	74.54
1973 Aug. 10	11:35:15	14:43:56	283	−17.90	321.10	7.02
1973 Aug. 15	08:26:24	15:21:51	633	−17.25	227.50	328.62
1973 Aug. 22	08:42:36	15:02:29	243	−16.40	165.10	257.58
1973 Aug. 25	08:01:45	13:19:50	447	−16.08	126.86	204.30
1973 Aug. 28	07:42:13	14:48:42	615	−15.78	93.90	197.74
1973 Sep. 01	07:04:49	12:46:29	395	−15.44	47.34	130.54
1973 Sep. 04	08:04:32	15:25:22	645	−15.22	33.90	141.26
1973 Sep. 08	07:17:17	14:57:46	680	−15.00	345.26	97.42
1973 Sep. 12	06:00:23	11:59:00	492	−14.86	289.58	16.94
1973 Sep. 15	06:39:22	11:55:16	460	−14.80	271.50	348.47
1973 Sep. 22	06:05:19	07:19:31	114	−14.88	199.34	217.42
1973 Sep. 29	05:18:01	09:26:47	227	−15.26	124.62	185.26
1973 Oct. 05	04:41:29	13:52:03	779	−15.83	62.06	196.30

Table 5 (continued)

Date	Start	End	No.	Latitude	Longitude coverage	
1973 Oct. 06	03:44:08	11:33:59	704	−15.94	39.18	153.74
1973 Oct. 12	05:34:39	11:27:38	523	−16.72	12.94	99.02
1973 Oct. 15	05:52:40	11:22:01	503	−17.16	350.86	71.18
1973 Oct. 19	07:04:54	11:31:54	275	−17.78	333.26	38.37
1973 Oct. 22	05:15:15	11:37:43	571	−18.26	280.13	13.42
1973 Oct. 23	04:28:38	05:21:07	81	−18.39	259.97	272.77
1973 Oct. 24	07:56:33	11:33:42	332	−18.59	301.89	354.86
1973 Nov. 01	04:32:02	11:26:41	618	−19.82	181.58	282.70
1973 Nov. 05	09:01:46	12:17:58	300	−20.40	211.98	259.82
1973 Nov. 11	03:19:36	11:21:19	735	−21.12	75.17	192.61
1973 Nov. 12	02:25:03	11:33:04	836	−21.23	52.94	186.54
1973 Nov. 24	02:26:11	10:24:12	707	−22.18	304.94	61.42
1975 Oct. 13	07:26:35	18:27:48	248	2.84	171.45	332.45
1975 Nov. 16	07:32:52	15:01:11	684	2.86	220.27	329.55
1975 Nov. 17	06:30:37	14:16:39	684	2.75	196.11	309.71
1975 Dec. 20	05:24:09	06:45:30	125	−3.26	248.27	268.11
1975 Dec. 27	02:49:51	03:12:24	12	−4.46	148.95	154.45
1975 Dec. 28	03:24:03	09:43:23	153	−4.64	148.45	240.95
1975 Dec. 30	06:39:44	10:54:00	118	−4.94	178.46	240.46
1976 Jan. 11	07:01:01	09:43:04	46	−6.10	76.44	115.94
1976 Jan. 15	06:51:14	09:00:29	63	−6.27	37.94	69.44
1976 Jan. 17	03:41:23	08:59:49	450	−6.31	333.54	51.14
1976 Jan. 23	03:47:50	10:54:41	208	−6.28	280.45	24.45
1976 Jan. 30	04:05:22	10:04:34	151	−5.99	220.45	307.95
1976 Feb. 01	04:29:24	09:35:15	133	−5.86	207.84	282.34
1976 Feb. 02	02:03:00	09:07:55	156	−5.79	162.94	266.44
1976 Feb. 17	00:21:02	04:21:17	84	−4.17	358.44	56.94
1976 Feb. 25	03:20:17	07:18:31	86	−2.96	326.94	24.94
1976 Mar. 13	22:19:30	23:31:24	7	0.28	83.44	100.94
1976 Mar. 30	22:30:30	23:50:38	34	3.88	284.19	303.69
1976 Mar. 31	23:01:32	00:31:56	41	4.10	282.19	304.19
1976 Apr. 01	23:34:39	00:56:50	31	4.33	280.69	300.70
1978 Jan. 23	07:16:13	09:13:33	149	12.11	110.83	139.45
1978 Jan. 28	11:05:35	13:48:40	55	11.37	123.07	162.85
1978 Jan. 30	06:41:37	08:52:58	179	11.12	41.17	73.21
1978 Jan. 31	05:01:45	12:23:47	219	11.00	8.05	115.86
1978 Feb. 01	07:37:18	13:28:35	376	10.86	37.21	122.89
1978 Feb. 07	06:47:25	12:57:56	460	10.20	332.22	62.58
1978 Feb. 14	07:25:05	11:49:23	311	9.66	279.30	343.74
1978 Feb. 18	09:26:39	11:41:02	160	9.48	273.18	305.94
1978 Feb. 28	03:55:20	10:46:45	215	9.41	102.18	202.44
1978 Mar. 05	23:33:05	09:56:37	688	9.61	343.58	135.50
1978 Mar. 12	05:07:01	10:05:32	206	9.98	9.82	82.54
1978 Mar. 14	23:32:17	09:26:26	492	10.21	260.59	45.31

by the ephemerides. The DSS 14 observations of Mars for 1971, listed in the table, are those which were delivered initially; they represent smoothed mean values of the set of finely-spaced points which were delivered subsequently. Table 5 gives a summary of the finely-spaced DSS 14 radar observations of Mars covering the oppositions of 1971 through 1978.

The strength of a radar echo from a planet's surface varies as the inverse fourth power of the distance. Up to the time of creation of DE 118 (1981), high-quality planetary radar-ranging observations were confined to times when the distances were less than about 1 AU. At these distances, the precision of the ranging observations was seen to have been about 100 m. However, for ephemeris measurements, variations in the topography of the planet's surface introduce random-like variations into the observations which tend to dominate the uncertainties. In order to make full use of the inherent accuracy of the radar measurements, one must introduce special methods to account for these topographical signatures.

4.2. *Mars radar closure points*

In the past, a typical day's observing run for Mars at the 85 m antenna at Goldstone, CA, would last several hours, during which time a few hundred observations would be made, each point separated from the next by about 40 s. Due mainly to the rotation of Mars, these points cover 14°.6 h^{-1} in martian longitude while being at a nearly constant latitude.

If the planetary latitude from which the signal is reflected is nearly the same for two different days, and if the tracks overlap in longitude, then "closure" occurs. Since the same topographical features are observed during each day, the uncertainty introduced by the topography may be eliminated by subtracting the observations taken on one day from those taken of the same terrain on the other day. The remaining differences are then due only to the drift in the ephemeris between the two days. The mean difference, averaged over all points within a pair of overlapping days, is termed a "closure point". It is a differential ephemeris measurement, free from topographical considerations.

The closure points used for the JPL ephemerides were those for which the latitudes between days differed by no more than 1° and for which there were at least 15 individual points aligned in longitude. Typically, when the latitude difference was within 0°.1, the scatter about the mean difference was less than 1 μs (round-trip, equivalent to about 150 m). This scatter degrades by a factor of two or three when the latitude difference approaches a full degree. Table 6 presents a summary of the numbers of closure points used in each ephemeris and from which opposition of Mars they have come.

4.3. *Mars radar/occultation point comparison*

The radio signal from the Mariner 9 orbiting spacecraft was periodically extinguished whenever the spacecraft disappeared behind the planet as seen from Earth. Timings of the extinctions and of the subsequent reappearances, coupled with accurate knowledge of the spacecraft's planet-centered ephemerides, allow one to determine the radius of the planet at the points on the surface which intersect the signal. These observations are discussed by Kliore et al. (1972). Associated with each observation is the longitude and latitude of the point on Mars where the signal was extinguished, as well as the derived altitude of the point above the center of mass of the planet.

Many of the radar ranging measurements of Mars have passed close to a number of the points on the surface which have been measured by the occultation timings. It is therefore possible to compare the results of the two methods, and since the occultation measurements are virtually free from planetary ephemeris error, the difference in results can be attributed mainly to errors in the planetary ephemeris of Mars. However, associated with each point is also the possibility of the occultation measurement being in error, due mainly to uncertainties of the orbiter's ephemeris about the planet. Also, if the radar point does not coincide exactly with the occultation point's position on the surface, a slope on the surface will introduce an additional discrepancy. Finally, an occultation will be caused by the highest nearby surface feature while a radar measurement tends to represent the mean of the topography throughout the region from which it is reflected. The accounting for these features during the incorporation of the data into the ephemerides is discussed next.

The ranging data to Mars that is listed in Tables 4 and 5 were scanned to locate any points which were reflected off the surface within 2° of the positions of any of the Mariner 9 occultation points. There were 45 occultation points for which this occurred. These 45 points were then located on the series of martian topographical maps available from the U.S. Geological Survey. For each point, the following quantitative measures describing the surrounding terrain were estimated: C, the compass direction of the uphill slope (measured clockwise from north); σ_C, the uncertainty of C; $1/S$, where S is the slope in km $\deg^{-1}$; $\sigma_{1/S}$, the uncertainty of $1/S$; T, the smoothness of the terrain and P, the radius of a circle surrounding the point within which one could safely extrapolate the general slope.

The Mars radar points were then selected which fell within one of the extrapolation areas around the occultation points. There were 2755 such points. For each of these radar points, an observation equation was formed:

$$O_i - R_j = -\Delta O_i - (Y_i + \Delta Y_i)\Delta\lambda - (Z_i + \Delta Z_i)\Delta\phi + \sum_k \delta R_j/\delta q_k \Delta q_k$$

where O_i is the i-th occultation measurement of the planet's radius, R_j is the j-th radar measurement (ephemeris distance minus observed distance); ΔO_i, a correction to O_i (to be estimated); Y_i and Z_i, the slopes in longitude and latitude at the i-th point (derived from C_i and S_i); ΔY_i and ΔZ_i, the corrections to the longitude and latitude slopes (to be estimated); $\Delta\lambda$ and $\Delta\phi$, the differences in longitude and latitude between the radar point and the occultation point. The vector q represents the set of solution parameters relevant to the correction of the ephemerides. Each equation was normalized to unit weight by dividing by the factor,

$$\sigma = \sqrt{(\sigma O_i)^2 + (\sigma R_j)^2},$$

where the uncertainty of the occultation measurement, σO_i, in order to account for the terrain, was taken as 1 km/$\sqrt{T}$ and the uncertainty for the radar measurement, σR_j, was taken as 150 m.

For each of the 45 occultation points, there were three additional equations in order to constrain the three adjustment parameters, ΔO_i, ΔY_i and ΔZ_i, using σO_i and a priori uncertainties derived from σ_C and σ_S. The 45 sets of three parameters were estimated (implicitly) in the solution for DE 108.

The radar/occultation point comparison observations were used in the solution for DE 108 only; they were later abandoned

Table 6. Closure points from DSS 14 used in the JPL ephemerides

Overlapping years	Number of overlapping pairs				
	DE 96	DE 102	DE 108	DE 111	DE 118
1971/1971	106	106	106	0	0
1971/1973	121	121	121	0	0
1973/1973	64	64	64	0	0
1975/1976	0	15	15	0	0
1978/1978	0	0	15	0	0
Total	291	306	321	0	0

Table 7. Mars radar/occultation point comparison

Occ. pt. (1)	No. (2)	No. obs. (3)	Lat. [deg] (4)	Long. [deg] (5)	Alt. [km] (6)	Uphill [deg] (7)	(8)	1/slp. [deg km^{-1}] (9)	(10)	Rad [deg] (11)	Sm. 1–9 (12)	Alt. corr. [km] (13)	(14)	σ [km] (15)
35	N	29	−22.8	175.2	3394.75	225	± 60	7	± 5	2.0	7	0.42	±0.08	±0.42
36	N	25	−22.1	350.3	3394.92	135	30	10	2	2.0	5	0.75	0.08	0.89
37	N	70	−21.5	165.4	3394.88	225	45	5	3	2.0	7	1.15	0.07	1.09
38	N	35	−20.9	340.7	3395.35	150	30	7	3	2.0	8	0.24	0.04	0.38
39	N	86	−20.3	155.9	3394.38	200	45	7	4	2.0	6	0.45	0.04	0.78
40	N	76	−19.7	331.0	3396.44	135	30	3	1	2.0	6	0.48	0.08	0.46
41	N	46	−19.0	146.2	3396.21	70	45	8	4	2.0	5	1.60	0.11	1.26
42	N	156	−18.4	321.4	3396.99	310	90	20	20	2.0	4	0.35	0.02	0.32
43	N	115	−17.7	136.6	3396.91	70	20	6	2	2.0	9	0.32	0.03	0.68
44	N	223	−17.1	311.8	3397.31	320	30	5	2	2.0	4	0.35	0.02	0.54
45	N	283	−16.4	127.0	3399.53	50	15	2	0	2.0	9	0.41	0.02	0.81
46	N	75	−15.7	302.1	3396.78	320	180	20	20	1.5	5	0.36	0.05	0.83
47	N	208	−14.9	117.3	3402.59	325	20	2	0	2.0	8	−0.07	0.02	0.79
48	N	147	−14.1	292.5	3397.24	200	90	6	4	2.0	4	0.80	0.05	0.92
49	N	53	−13.4	107.6	3402.05	70	40	10	8	2.0	7	0.33	0.05	0.51
50	N	43	−12.6	282.7	3396.96	60	20	2	1	2.0	5	1.11	0.22	0.86
56	N	8	−7.6	253.0	3397.08	250	60	20	10	2.0	7	0.75	0.15	0.98
57	N	1	−6.7	68.0	3398.92	80	180	4	4	0.5	1	−0.50	0.53	0.73
58	N	16	−5.8	243.0	3396.61	225	90	15	8	2.0	6	0.69	0.08	0.78
59	N	8	−4.8	58.1	3397.89	290	30	3	1	2.0	6	0.51	0.14	0.81
60	N	15	−3.8	233.2	3397.07	225	30	4	1	2.0	6	0.08	0.06	0.44
61	N	3	−2.8	48.3	3396.84	250	45	6	3	2.0	4	0.47	0.24	0.85
67	N	3	3.8	19.7	3394.26	120	180	8	8	2.0	3	0.78	0.31	0.97
68	N	2	5.1	195.1	3392.54	30	60	10	10	2.0	9	0.24	0.16	0.73
69	N	2	6.4	10.6	3393.89	30	180	15	15	2.0	3	0.40	0.36	1.03
70	N	13	7.7	186.1	3392.28	0	60	20	20	2.0	9	1.26	0.16	1.43
71	N	45	9.2	1.7	3393.73	340	90	10	10	2.0	4	0.82	0.06	0.62
72	N	18	10.7	177.4	3391.72	330	45	12	4	2.0	7	1.00	0.15	1.50
73	N	5	12.4	353.2	3393.82	80	45	8	3	2.0	6	0.26	0.28	1.61
77	N	56	20.8	337.9	3393.14	70	45	15	5	2.0	7	1.18	0.06	1.14
78	N	15	24.0	155.0	3388.61	90	90	8	6	2.0	9	−0.23	0.10	0.61
358	X	166	−16.1	305.5	3398.50	210	180	20	20	2.0	5	2.04	0.03	0.88
420	N	57	22.4	172.4	3390.40	290	45	10	3	2.0	7	1.53	0.12	1.65
422	N	37	19.5	164.0	3391.10	280	45	15	10	2.0	9	1.43	0.07	0.89
428	N	36	10.5	139.2	3394.00	120	45	10	3	2.0	7	−0.07	0.09	1.37
438	N	7	−5.4	98.0	3407.10	270	60	3	3	2.0	3	2.74	0.62	2.18
442	N	43	−12.7	81.5	3401.30	300	45	10	6	2.0	8	2.14	0.13	1.07
446	N	40	−21.0	65.5	3399.00	315	45	7	3	2.0	5	2.03	0.08	1.08
662	X	38	−21.4	254.3	3395.51	300	180	20	20	2.0	3	−0.36	0.06	0.35
666	X	146	−18.3	236.7	3396.18	150	90	10	10	2.0	3	0.18	0.02	0.43
670	X	143	−15.5	218.6	3394.19	200	180	20	15	2.0	4	−1.79	0.03	0.57
672	X	168	−14.1	209.4	3395.33	200	30	4	2	2.0	6	−0.27	0.03	0.48
674	X	10	−12.7	200.0	3393.88	225	30	3	1	2.0	7	−0.19	0.10	0.44

Notes: The columns contain (1) orbit number, (2) entry (N) or exit (X), (3) number of radar points, (4–5) latitude and longitude, (6) occultation altitude, (7–8) compass direction and s.d. of the uphill slope, (9–10) inverse of the slope and s.d., (11) radius of circle within which slope data may be extrapolated, (12) smoothness (1: rough; 9: smooth), (13–14) altitude correction and formal s.d. to altitude (subsequently derived using DE 118), (15) s.d. of a single comparison point

1990A&A...233..252S

in favor of the highly accurate Viking Lander range data (discussed below); they are being resurrected for use in future ephemerides along with similar occultation points from the Viking mission.

Table 7 gives the pertinent data of the Mars occultation points. Also in the table are corrections to the occultation values, ΔO_i, derived subsequently, using DE 118. This latter determination is not exactly the same as the original pre-DE 108 set of observation equations, since slightly different selection criteria have been used. However, the table closely approximates the data used in DE 108.

5. Spacecraft range points

There were four different sets of spacecraft data used in the ephemeris solutions up to and including that for DE 118. All of these contained range information only. They are 1) normal points from the Pioneer Missions to Jupiter, 2) normal points to the Mariner 9 orbiter of Mars, 3) normal points to the Viking orbiters of Mars and 4) actual range measurements to the Viking Landers on the surface of Mars.

Normal points represent modified distance measurements. The original round-trip range and doppler measurements have been reduced using the JPL Orbit Determination Program (Moyer, 1971). This reduction is an adjustment for many various parameters, including the spacecraft orbit, the planet's mass and gravity field, etc. As such, the resultant range residuals represent derived corrections to the nominal planetary ephemeris used in the reduction. These residuals are then used to correct that nominal ephemeris.

5.1. Pioneer normal range points to Jupiter

There was one normal range point from each of the two Pioneer encounters of Jupiter. The major uncertainty for each point comes from the uncertainties in the determination of the spacecrafts' orbit with respect to the center of mass of the planet. The observations are listed in Table 8 where the residual has been added to the geometric (instantaneous Earth–planet) range in order to form a pseudo "observed" range point.

Table 8. Pioneer normal range points to Jupiter. These values represent geometric (instantaneous) distances from the center of mass of the earth to the barycenter of the planetary system

Pioneer 10	1973 Dec. 04 0 hr, E.T.	(2442020.50)	2754747323 ± 40 μs
Pioneer 11	1974 Dec. 03 0 hr, E.T.	(2442384.50)	2439811990 ± 10 μs

5.2. Mariner 9 normal range points to Mars

During its lifetime, the onboard range transponder of the Mariner 9 Orbiter allowed accurate range measurements to that spacecraft in its orbit about Mars. These data exist in four sets according to their proximity to the Martian solar conjunction (JD 2441568) when the 2300-MHz ranging signal passed within 4 solar radii of the Sun at the heliographic latitude of +79. The major uncertainties of these points when near conjunction are due to uncertainties in the densities of the ionized electrons in the solar corona through which the signal passed. At times away from conjunction, orbital uncertainties predominate. The data have been corrected for the solar corona using the formula given in Sect. 3.

Table 9. Mariner 9 Orbiter normal range points to Mars. The four sets of points are grouped according to the proximity in time to the solar conjunction of Mars (JD 2441568)

Date	σ[μs]	DE 96	DE 102	DE 108	DE 111	DE 118
2441272–2441361	0.25	77	77	77	77	77
2441389–2441540	0.29	81	81	81	81	81
2441541–2441555, 2441577–2441602	0.78	487	487	487	487	487
2441556–2441575	2.50	159	158	158	158	0
Total		804	803	803	803	645

5.3. Viking Orbiter normal range points to Mars

Similar to the Mariner 9 range points, ranging data from the Viking Orbiters were taken over the lifetimes of the two spacecraft. A major difference, however, was that for the Viking orbiters, the ranging was done simultaneously in two separate frequencies, 2300 MHz and 8300 MHz. It was therefore possible to calibrate the time-delay caused by the solar corona, since the delay is frequency-dependent. The times and amounts of these orbiter ranges are given in Table 10. Even though the data continued to be taken until the loss of the orbiters in 1980, the data were used directly in DE 102 and DE 108 only. They were superceded by the more accurate Viking Lander data beginning with DE 108. The major uncertainties in the orbiter data come from uncertainties in the spacecrafts' orbits.

5.4. Viking Lander range points to Mars

The most accurate of all planetary position data are the two-way ranging measurements taken of the Viking Landers on the

Table 10. Viking range points from the Orbiters and the Landers

	Date	DE 96	DE 102	DE 108	DE 111	DE 118
Obtr. No. 1	2442949–2443223	0	2498	2362	0	0
Obtr. No. 2	2442998–2443191	0	1965	530	0	0
	Total Orbiter points	0	4463	2892	0	0
Lndr. No. 1	2442980–2443242	0	0	87		
	2442980–2443843				587	
	2442980–2444054					683
Lndr. No. 2	2443026–2443201	0	0	60		
	2443026–2443417				78	78
	Total Lander points	0	0	147	665	761

Table 11. Totals of all observations in the ephemeris solutions

Type of observation	Number of observations				
	DE 96	DE 102	DE 108	DE 111	DE 118
Washington transits	37853	37853	38942	39396	39579
Saturn astrometry	0	0	0	4790	4790
Neptune, Pluto normal points	0	0	0	0	386
Radar ranging	5052	5052	1199	1307	1307
Mars radar closure	291	306	321	0	0
Mars radar/occ. pt. comparison	0	0	2890	0	0
Jupiter Pioneer normal points	2	2	2	2	2
Mariner 9 normal points	804	803	803	803	645
Viking Orbiter normal points	0	4463	2892	0	0
Viking Lander range points	0	0	147	665	761
Lunar laser ranging	0[a]	0[a]	0[a]	2531	2954
Total	44002	48479	47196	49494	50424

[a] Though the equations of motion for the Moon and planets were integrated simultaneously for all of the ephemerides, the solutions for the lunar parameters were done separately previous to DE 111

Table 12. The approximate a priori standard deviations of the various data types with which the observational equations were normalized

Transit data	Sun, Mercury, Venus	1″.0
	Jupiter, . . . , Neptune	0″.5
Saturn astrometry	Catalogue stars	0″.3
	Star and Satellite measures	0″.3
Neptune and Pluto normal points		0″.5
Radar ranging	observations before 1967	10 km
	after 1967	1.5 km
Mars radar closure		150–500 m
Mars radar/occultation point comparison		0.3–1.0 km
Pioneer normal range points		10 km
Mariner 9 normal range points		40–400 m
Viking Orbiter normal range points		40 m
Viking Lander ranges		6 m
Lunar laser ranging		18 cm

surface of Mars. Though only a single frequency (2300 MHz) was used, the solar corona could be calibrated by using the nearly simultaneous dual frequency measurements of the Viking Orbiters. Observations were made on the average of once per week, typically with about six range points per day. The residuals are seen to have a scatter of only 2–3 m about the mean of the day; the means for each day, however, show a scatter of about 6 m amongst themselves. Without orbital uncertainties being present, the dominating contributors to these residuals come from three sources: the calibration of the solar corona, the calibration of the inherent time-delays in the tracking station antennas (done before and after each pass) and the calibration of the transponders of the landers themselves (done before launch).

The range data were reduced using the formulae given in Sect. 3, with the reflection point on the planet's surface being replaced by the location of the lander on Mars. For this, one needs the martian coordinates of the landers as well as a set of angles used to express the orientation of Mars within the ephemeris reference frame. The position of the lander, expressed in the

Table 13. Values of various constants used in the JPL ephemerides. The values in brackets were adopted for the solution, not solved-for. The rounded values for some of the DE 118 parameters are the result of the two-step process described in the text; they are denoted by asterisks (*)

Ephemeris	DE 96	DE 102	DE 108	DE 111	DE 118
km/AU: 149597000+					
	871.410558	870.683518	870.705416	870.652948	870.66*
GM(Earth)/*GM*(Moon)					
	[81.3007]	[81.3007]	81.300492	[81.300587]	[81.300587]
Sun–planet mass ratios					
(Mer)	[6023600]	[6023600]	[6023600]	[6023600]	[6023600]
(Ven)	[408523.5]	[408523.5]	[408523.5]	[408523.5]	[408523.5]
(EMB)	[328900.53]	[328900.53]	[328900.53]	[328900.53]	[328900.55]
(Mar)	[3098710]	[3098710]	[3098710]	[3098710]	[3098710]
(Jup)	[1047.355]	[1047.355]	1047.354	1047.350575	1047.350*
(Sat)	[3498.5]	[3498.5]	3498.0	3498.0158	3498.0*
(Ura)	[22869]	[22869]	22945	22960.85	22960*
(Nep)	[19314]	[19314]	19091	19122.05	[19314]
(Plu)	[3000000]	[3000000]	[3000000]	[200000000]	[130000000]
Asteroid *GM* [*GM*(Sun) = 0.01720209895^2]					
Ceres	[0.1746–012]	[0.1746–012]	[0.1746–012]	[0.1746–012]	[0.1746–012]
Pallas	[0.3847–013]	[0.3847–013]	[0.3850–013]	[0.3850–013]	[0.3200–013]
Vesta	[0·3551–013]	[0.3551–013]	[0.3550–013]	[0.3550–013]	[0.4080–013]
Iris	[0.1580–014]	[0.1600–014]	[0.1600–014]	[0.1600–014]	
Bamberga	[0.2576–014]	[0.2600–014]	[0.2600–014]	[0.2600–014]	
Planetary radii [km]					
Mercury	2440.122	2439.958	2440.019	2439.988	2439.990*
Venus	6052.058	6051.813	6051.824	6051.803	6051.813*
Mars	3397.515	3396.644	[3397.515]	[3397.515]	[3397.515]
Catalogue drift [$''\,\mathrm{cty}^{-1}$]					
Δk	−1.191	−0.760	−0·566	−0·783	[−0.266]
Δn	0.148	0.289	0.424	[0.438]	[0.438]

Catalogue offset [DE 118 only]	W 10	W 20	W 25	W 40	W 150	W 250	W 350	W 450	W 550	Wcir
$\Delta\alpha$	0″.37	−0″.01	0″.08	−0″.03	−0″.03	−0″.10	−0″.02	−0″.11	−0″.09	−0″.23
$\Delta\delta$	0″.30	−0″.03	0″.08	−0″.02	0″.09	0″.09	−0″.02	−0″.00	−0″.03	−0″.02

	DE 96	DE 102	DE 108	DE 111	DE 118
Limb biases					
Δl_α (Mer)	0″.09				
Δl_δ (Mer)	0″.04				
Δl_α (Ven)	0″.53				
Δl_δ (Ven)	0″.23				
Phase correction					
C0		−0.03	−0.03	−0.03	−0.03
C1		0.45	0.43	0.43	0.39
C2		0.39	0.43	0.42	0.47
C3		−0.24	−0.26	−0.25	−0.26
L0		−0″.13	−0″.17	−0″.13	−0″.15
L1		2″.69	2″.88	2″.73	2″.81
L2		−2″.25	−2″.50	−2″.29	−2″.45
L3		0″.64	0″.74	0″.66	0″.73
B4		0″.42	0″.41	0″.40	0″.40
B5		0″.98	0″.86	0″.94	0″.97
B6		0″.84	0″.87	0″.88	0″.74
B7		0″.39	0″.42	0″.44	0″.20
B8		1″.21	1″.33	1″.23	0″.93

Table 13 (continued)

Ephemeris	DE 96	DE 102	DE 108	DE 111	DE 118
Day correction					
A1	0″.31	−0″.12	−0″.24	−0″.26	0″.27
A2	0″.21	0″.20	0″.20	0″.23	0″.38
A3	−0″.27	0″.18	0″.16	0″.38	
D1	−0″.53	−0″.36	−0″.39	−0″.32	0″.15
D2	−1″.41	−1″.38	−1″.35	−1″.51	−1″.13
D3	0″.68	0″.50	0″.54	0″.47	
Solar corona constant for Mariner 9					
$A\,10^8$	[1.3]	[1.3]	[1.3]	1.23	1.22
$B\,10^6$	[0.5]	[0.5]	[0.5]		
$a\,10^6$				0.36	0.44
$b\,10^6$				0.56	0.44
Mars orientation parameters					
V [deg]			[328.70742325]	[328.70742325]	[328.70742325]
$\dot{V}$ [deg d^{-1}]			350.89197887	350.89198171	350.89199047
I_q [deg]			25.17797755	25.20058362	25.18084149
$\dot{I}_q$ [deg cty^{-1}]			[0.01221]	−0.042099	0.030012
Ω_q [deg]			35.24205662	35.24959294	35.33715552
$\dot{\Omega}_q$ [deg cty^{-1}]			[0.1048]	0.201552	−0.118037
I [deg]			[1.85]	[1.85]	[1.85]
$\dot{I}$ [deg cty^{-1}]			[−0.0082]	[−0.0082]	[−0.0082]
Ω [deg]			[49.17193]	[49.17193]	[49.17193]
$\dot{\Omega}$ [deg cty^{-1}]			[−0.294700]	[−0.294700]	[−0.294700]
ε [deg]			[23.445789]	[23.445789]	[23.445789]
$\dot{\varepsilon}$ [deg cty^{-1}]			[−0.013019]	[−0.013019]	[−0.013019]
Viking Lander coordinate					
u_1 [km]			3136.527	3136.513	3136.515
v_1 [km]			1284.547	1284.847	1284.587
λ_1 [deg]			311.9197975	311.8904103	311.8027066
u_2 [km]			2277.411	2277.381	2277.374
v_2 [km]			2500.193	2500.444	2500.184
λ_2 [deg]			134.1512420	134.1221807	134.0343054
Earth station location				(5 par's)	(17 par's)
LLR parameter				(39 par's)	(33 par's)
Total number of explicit parameter	DE 96	DE 102	DE 108	DE 111	DE 118
	64	71	86	133	165

Table 14. The formal standard deviations of the solution parameters

	DE 96	DE 102	DE 108	DE 111	DE 118
Planetary set III orbital corrections ($\Delta l + \Delta r$)					
Mer	0″.009	0″.008	0″.008	0″.007	0″.01
Ven	0″.009	0″.008	0″.008	0″.007	0″.01
EMB	0″.009	0″.008	0″.008	0″.007	0″.01
Mar	0″.009	0″.008	0″.008	0″.007	0″.01
Jup	0″.02	0″.02	0″.02	0″.01	0″.02
Sat	0″.02	0″.03	0″.02	0″.02	0″.02
Ura	0″.07	0″.07	0″.06	0″.06	0″.05
Nep	0″.3	0″.4	0″.3	0″.2	0″.07
Plu					2″.0
Moon				0″.006	0″.01

Table 14 (continued)

	DE 96	DE 102	DE 108	DE 111	DE 118
Δp					
Mer	0.″01	0.″005	0.″01	0.″005	0.″007
Ven	0.″01	0.″001	0.″01	0.″003	0.″005
EMB	0.″01		0.″01	0.″003	0.″005
Mar	0.″01	0.″0004	0.″01	0.″002	0.″003
Jup	0.″02	0.″02	0.″02	0.″02	0.″02
Sat	0.″02	0.″02	0.″02	0.″02	0.″02
Ura	0.″02	0.″02	0.″02	0.″02	0.″02
Nep	0.″02	0.″02	0.″02	0.″02	0.″02
Plu					0.″1
Moon				0.″004	0.″006
Δq					
Mer	0.″01	0.″004	0.″01	0.″004	0.″004
Ven	0.″01	0.″001	0.″01	0.″003	0.″003
EMB	0.″01		0.″01	0.″002	0.″002
Mar	0.″01	0.″0005	0.″01	0.″003	0.″005
Jup	0.″02	0.″02	0.″02	0.″01	0.″02
Sat	0.″02	0.″02	0.″02	0.″02	0.″01
Ura	0.″02	0.″02	0.″02	0.″02	0.″02
Nep	0.″02	0.″02	0.″02	0.″02	0.″02
Plu					0.″3
Moon				0.″002	0.″002
$e\Delta r$					
Mer	0.″002	0.″002	0.″002	0.″0001	0.″002
Ven	0.″0001	0.″00009	0.″00009	0.″001	0.″0001
EMB	0.″0001	0.″0001	0.″0001	0.″00008	0.″0002
Mar	0.″0009	0.″0008	0.″0007	0.″0006	0.″001
Jup	0.″004	0.″004	0.″004	0.″004	0.″004
Sat	0.″009	0.″009	0.″009	0.″009	0.″008
Ura	0.″03	0.″03	0.″03	0.″03	0.″03
Nep	0.″2	0.″2	0.″1	0.″1	0.″03
Plu					2.″0
Moon				0.″0005	0.″0008
$\Delta a/a$					
Mer	0.″000008	0.″000007	0.″000006	0.″00001	0.″000006
Ven	0.″00002	0.″000006	0.″000006	0.″000006	0.″000004
EMB	0.″00002	0.″000002	0.″000009	0.″000006	0.″000002
Mar	0.″00004	0.″000004	0.″00002	0.″00002	0.″000006
Jup	0.″0008	0.″0008	0.″0009	0.″0009	0.″0007
Sat	0.″003	0.″003	0.″003	0.″003	0.″002
Ura	0.″03	0.″03	0.″02	0.″02	0.″02
Nep	0.″2	0.″2	0.″2	0.″2	0.″03
Plu					2.″0
Moon				0.″00001	0.″000009
Δe					
Mer	0.″0003	0.″0003	0.″0003	0.″0002	0.″0003
Ven	0.″00009	0.″00009	0.″00008	0.″00008	0.″00008
EMB	0.″00001	0.″000009	0.″00001	0.″00002	0.″000004
Mar	0.″00002	0.″000009	0.″00001	0.″00008	0.″000008
Jup	0.″004	0.″005	0.″005	0.″004	0.″004
Sat	0.″009	0.″01	0.″01	0.″01	0.″008
Ura	0.″01	0.″01	0.″01	0.″01	0.″01
Nep	0.″1	0.″1	0.″1	0.″1	0.″02
Plu					0.″8
Moon				0.″00001	0.″000008
km AU^{-1}	0.03	0.005	0.008	0.01	0.002
GM(Earth)/GM(Moon)			0.00002		

Table 14 (continued)

	DE 96	DE 102	DE 108	DE 111	DE 118
$GM(\text{Sun})/GM(\text{Jup})$			0.0002	0.0005	0.0001
$GM(\text{Sun})/GM(\text{Sat})$			0.04	0.05	0.01
$GM(\text{Sun})/GM(\text{Ura})$			15	15	6
$GM(\text{Sun})/GM(\text{Nep})$			60	60	
Planetary radii [km]					
$R(\text{Mer})$	0.1	0.1	0.1	0.09	0.09
$R(\text{Ven})$	0.06	0.07	0.06	0.06	0.06
$R(\text{Mar})$	0.04	0.03			
Catalog drift parameter [$''\,\text{cty}^{-1}$]					
Δk	0.05	0.05	0.04	0.04	
Δn	0.04	0.04	0.04		
Catalog offset					
W550, . . . , Wcir					$0\farcs02$
Limb bias					
$\Delta l_\alpha(\text{Mer})$	$0\farcs3$				
$\Delta l_\delta(\text{Mer})$	$0\farcs4$				
$\Delta l_\alpha(\text{Ven})$	$0\farcs02$				
$\Delta l_\delta(\text{Ven})$	$0\farcs02$				
Phase correction					
C0		0.02	0.02	0.02	0.02
C1		0.1	0.09	0.09	0.09
C2		0.1	0.1	0.1	0.1
C3		0.04	0.04	0.04	0.04
L0		$0\farcs08$	$0\farcs07$	$0\farcs07$	$0\farcs07$
L1		$0\farcs3$	$0\farcs3$	$0\farcs3$	$0\farcs3$
L2		$0\farcs4$	$0\farcs4$	$0\farcs4$	$0\farcs4$
L3		$0\farcs2$	$0\farcs1$	$0\farcs1$	$0\farcs1$
B4		$0\farcs03$	$0\farcs02$	$0\farcs02$	$0\farcs02$
B5		$0\farcs1$	$0\farcs08$	$0\farcs08$	$0\farcs08$
B6		$0\farcs2$	$0\farcs2$	$0\farcs1$	$0\farcs1$
B7		$0\farcs4$	$0\farcs3$	$0\farcs3$	$0\farcs3$
B8		$0\farcs6$	$0\farcs5$	$0\farcs5$	$0\farcs5$
Day correction					
A1	$0\farcs2$	$0\farcs2$	$0\farcs2$	$0\farcs2$	$0\farcs05$
A2	$0\farcs08$	$0\farcs09$	$0\farcs09$	$0\farcs07$	$0\farcs2$
A3	$0\farcs3$	$0\farcs3$	$0\farcs3$	$0\farcs2$	
D1	$0\farcs2$	$0\farcs3$	$0\farcs3$	$0\farcs2$	$0\farcs05$
D2	$0\farcs08$	$0\farcs08$	$0\farcs09$	$0\farcs07$	$0\farcs2$
D3	$0\farcs3$	$0\farcs3$	$0\farcs3$	$0\farcs2$	
Solar corona parameter					
$A\,10^8$				0.4	0.03
$a\,10^6$				0.2	0.02
$b\,10^6$				0.2	0.03
Mars orientation parameter					
$\dot{V}$ [$\text{deg}\,\text{d}^{-1}$]			0.000008	0.000001	0.0000003
I [deg]			0.02	0.005	0.002
$\dot{I}_q$ [$\text{deg}\,\text{cty}^{-1}$]				0.02	0.006
Ω_q [deg]			0.03	0.01	0.003
$\dot{\Omega}_q$ [$\text{deg}\,\text{cty}^{-1}$]				0.04	0.01

Table 14 (continued)

	DE 96	DE 102	DE 108	DE 111	DE 118
Viking Lander coordinate					
u_1 [km]			0.01	0.003	0.001
v_1 [km]			0.02	0.2	0.02
λ_1 [deg]			0.02	0.01	0.003
u_2 [km]			0.01	0.004	0.002
v_2 [km]			0.04	0.2	0.02
λ_2 [deg]			0.02	0.01	0.003

frame of the ephemeris (equator and equinox), is given by

$$\boldsymbol{r} = R_x(-\varepsilon)R_z(-\Omega)R_x(-I)R_z(-\Omega'_q)R_x(-I'_q)R_z(-V')\boldsymbol{r}_0$$

where the basic angles are expressed as linear functions in time, thereby allowing orbital and planetary precession. The angle ε is the obliquity of the ecliptic; Ω and I, the node and inclination of the mean martian orbit upon the ecliptic; Ω_q and I_q, the mean node and inclination of the martian equator upon the mean orbit; V, the longitude of the martian prime meridian measured along the equator from the intersection of the orbit. The Mars-fixed coordinates of the lander are given by r_0 and the primed angles are

$$\Omega'_q = \Omega_q - \Delta\psi_{\mathrm{M}}, \quad I'_q = I_q + \Delta\varepsilon_{\mathrm{M}} \quad \text{and} \quad V' = V + \Delta\psi_{\mathrm{M}} \cos I_q,$$

where $\Delta\psi_{\mathrm{M}}$ and $\Delta\varepsilon_{\mathrm{M}}$ express the nutation of Mars, computed from the formulation of Lyttleton et al. (1979). The Mars-fixed coordinates of the lander are computed from the cylindrical coordinates,

$$r_0^{\mathrm{T}} = [u \cos\lambda, u \sin\lambda, v]^{\mathrm{T}}.$$

A summary for the Viking Lander ranging data is included in Table 10. The values for the parameters used in the reductions are included in Table 13. Standard values were adopted for ε, Ω and I; those for Ω_q, I_q and V, as well as the coordinates of the landers, were estimated in the least squares adjustments.

Though the estimated values of the orientation parameters provide reasonable fits to the observed data, they should not be interpreted as being definitive. They are highly correlated among themselves and are affected by remaining uncertainties in the masses of the asteroids. For a discussion of this latter problem, see Williams (1984).

6. Discussion

Characteristics of the various data sets often emerge during the processes of fitting the ephemerides to the data. A least squares adjustment involving a large variety of data is not a uniquely defined process. Inconsistencies between different data sets are nearly always present, at least on some level of accuracy; other determinations of certain parameters may be adopted; some parameters may be highly correlated; some parameters may be poorly determined; some data sets may be rendered non-essential by the acquisition of other, more accurate observations. This section discusses some of the decisions regarding the above questions. These are the unique features of the data processing which have been present in the adjustments to the JPL ephemerides.

6.1. *The observational equations*

Table 11 gives the number of observational equations for each different set of observational data used in the five ephemerides. Each observational equation was normalized by multiplying it by the factor, $1/\sigma_0$, where σ_0 is the a priori standard deviation of a single observation of a particular set of data. Previous experience in working with the various observational data sets has led to the knowledge of the individual accuracies of each type of observation. Table 12 presents the values of the a priori standard deviations of each of the sets of data; these are approximately equal to the root-mean-square (rms) of the post-fit residuals.

6.2. *Solution parameters*

Tables 13 and 14 give the values and the formal standard deviations, respectively, of the solution parameters of the five least-squares solutions used for adjusting the ephemerides. The adjustment of the orbital parameters is inherent in the values of the initial conditions of the ephemerides, listed in Table 15. A number of the parameters listed in Table 13 did not enter into the solution at all. A number of parameters were constrained to predetermined values, indicated in Table 13 by square brackets.

In addition to the parameters listed in the two tables, there was a number of other parameters which entered into the solutions, these implicitly. By taking advantage of the properties of sparse matrices, it is possible to eliminate from a set of normal equations, certain parameters which do not enter into any of the subsequent observational data. Such parameters are present in the final solution just as if they had been carried along explicitly; their presence, however, is not seen directly. Such was the case for the catalogue adjustments of the Saturn astrometrical data and for the slope and height adjustments of the radar/occultation comparisons.

In the solution for DE 102, the plane of the Earth–Moon's barycenter was left unchanged by not solving for the two relevant parameters, Δp and Δq. Subsequently, after the solution, the entire set of planetary initial conditions was rotated into alignment with the lunar solution (see Newhall et al., 1983). The lunar and planetary ephemerides in DE 96 and DE 108 were not specifically aligned with each other. The presence of the lunar data in the joint solutions for DE 111 and DE 118 automatically aligned the lunar and planetary ephemerides, making the procedure used in DE 102 unnecessary.

The solution for DE 118 was done in two steps. A preliminary solution was made involving all of the relevant solution parameters. The resulting values for certain specific parameters were then rounded off to their closest significant values. These

1990A&A...233..252S

Table 15. The initial conditions of the Ephemerides at JD 2440400.5 in [AU] and [AU d⁻¹]. Given are heliocentric coordinates for the planets, geocentric for the Moon and solar-system barycentric for the Sun

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
DE 96						
Mercury	0.3557922580285	−0.0955355879504	−0.0877134178162	0.0037087808508	0.0248497847800	0.0129268670834
Venus	0.6033503910337	−0.3559062740335	−0.1984879214940	0.0111569232028	0.0154889017564	0.0062751524204
E-Mbary	0.1036934360992	−0.9278340545910	−0.4023396941889	0.0168335341142	0.0015550298980	0.0006742293017
Mars	−0.1324779851142	−1.3269849183759	−0.6055541641333	0.0144822028777	0.0000753531675	−0.0003541207275
Jupiter	−5.3941958936061	−0.7709913063093	−0.1989103464060	0.0010056321327	−0.0065350248379	−0.0028281072475
Saturn	7.9482565922231	4.5071645178408	1.5199515065662	−0.0031591864994	0.0043662806909	0.0019419084081
Uranus	−18.2827495322406	−0.9583802950893	−0.1615738451072	0.0001713682705	−0.0037698523267	−0.0016541983147
Neptune	−16.3716953023856	−23.7616428640022	−9.3216250128726	0.0026228085969	−0.0015329315014	−0.0006940632445
Pluto	−30.4523499642980	−0.5324450661161	9.0594896804909	0.0002820486838	−0.0031521332523	−0.0010816447761
Moon	−0.0008357023040	−0.0019854403334	−0.0010832674180	0.0005987522064	−0.0001741533929	−0.0000884772995
Sun	0.0045277782503	0.0007233492806	0.0002437317358	−0.0000002839374	0.0000051830225	0.0000022313609
DE 102						
Mercury	0.3557925832924	−0.0955345485155	−0.0877132191455	0.0037087022239	0.0248497897092	0.0129268806051
Venus	0.6033515261042	−0.3559044873174	−0.1984876695292	0.0111568753206	0.0154889307388	0.0062751661931
E-Mbary	0.1036963194747	−0.9278335982481	−0.4023400078909	0.0168335292669	0.0015550772558	0.0006742408524
Mars	−0.1324738219138	−1.3269850541589	−0.6055547646852	0.0144822029825	0.0000753943351	−0.0003541110573
Jupiter	−5.3941932915539	−0.7710081200473	−0.1989150590738	0.0010056547485	−0.0065350206821	−0.0028281090999
Saturn	7.9482436367196	4.5071870232271	1.5199589946549	−0.0031592001390	0.0043662705224	0.0019419077867
Uranus	−18.2827508557988	−0.9584340318843	−0.1615867736876	0.0001713802650	−0.0037698507754	−0.0016541994628
Neptune	−16.3716335288293	−23.7617037839296	−9.3216533685651	0.0026228120284	−0.0015329244509	−0.0006940623979
Pluto	−30.4523559663809	−0.5325496136974	9.0594633601116	0.0002820600485	−0.0031521318753	−0.0010816458256
Moon	−0.0008356961442	−0.0019854419428	−0.0010832692002	0.0005987527434	−0.0001741517754	−0.0000884768645
Sun	0.0045277754174	0.0007233658692	0.0002437367327	−0.0000002839606	0.0000051830181	0.0000022313623
DE 108						
Mercury	0.3557922619520	−0.0955356148809	−0.0877133662244	0.0037087807034	0.0248497853751	0.0129268662331
Venus	0.6033503963396	−0.3559063243655	−0.1984878145912	0.0111569232817	0.0154889016095	0.0062751527033
E-Mbary	0.1036934336616	−0.9278340956795	−0.4023396051242	0.168335341028	0.0015550290754	0.0006742311449
Mars	−0.1324779672058	−1.3269849745181	−0.6055540351789	0.0144822029859	0.0000753526453	−0.0003541193151
Jupiter	−5.3941974664785	−0.7709876266693	−0.1989094810490	0.0010056275852	−0.0065350242925	−0.0028281069334
Saturn	7.9482629546463	4.5071580092641	1.5199489513674	−0.0031591823909	0.0043662826537	0.0019419085676
Uranus	−18.2827542886466	−0.9583689678224	−0.1615693323556	0.0001713662494	−0.0037698516271	−0.0016541979767
Neptune	−16.3717257216743	−23.7616519595999	−9.3216311374360	0.0026228066676	−0.0015329326646	−0.0006940637179
Pluto	−30.4523499642980	−0.5324450661161	9.0594896804909	0.0002820486838	−0.0031521332523	−0.0010816447761
Moon	−0.0008357023040	−0.0019854403334	−0.0010832674180	0.0005987522064	−0.0001741533929	−0.0000884772995
Sun	0.0045346842384	0.0007373522487	0.0002492669043	−0.0000002853662	0.0000051832214	0.0000022314609
DE 111						
Mercury	0.3557922056845	−0.0955357755691	−0.0877134189685	0.0037087951557	0.0248497768577	0.0129268784786
Venus	0.6033501891138	−0.3559065723936	−0.1984879999160	0.0111569322626	0.0154888921082	0.0062751601868
E-Mbary	0.1036928952888	−0.9278339564488	−0.4023400657643	0.0168335350008	0.0015550190939	0.0006742316402
Mars	−0.1324787391533	−1.3269846026524	−0.6055546820455	0.0144822030225	0.0000753444863	−0.0003541194740
Jupiter	−5.3941967321350	−0.7709883161678	−0.1989105167583	0.0010056287126	−0.0065350248876	−0.0028281071512
Saturn	7.9482628711392	4.5071579954748	1.5199497085039	−0.0031591826199	0.0043662827477	0.0019419076667
Uranus	−18.2827490039608	−0.9583689566581	−0.1615708365945	0.0001713683275	−0.0037698514721	−0.0016541989939
Neptune	−16.3717137327784	−23.7616447706433	−9.3216265451851	0.0026228074066	−0.0015329318513	−0.0006940632309
Pluto	−30.4523499642980	−0.5324450661161	9.0594896804909	0.0002820486838	−0.0031521332523	−0.0010816447761
Moon	−0.0008357035210	−0.0019854390373	−0.0010832688286	0.0005987521000	−0.0001741538297	−0.0000884771802
Sun	0.0045228034250	0.0007351654860	0.0002514542756	−0.0000002850583	0.0000051819763	0.0000022310117
DE 118						
Mercury	0.3557922218183	−0.0955357051533	−0.0877134311222	0.0037087906743	0.0248497775730	0.0129268783558
Venus	0.6033502530459	−0.3559064530483	−0.1984880201035	0.0111569294057	0.0154888943192	0.0062751597958
E-Mbary	0.1036930656779	−0.9278339358058	−0.4023400695644	0.0168335347145	0.0015550224233	0.0006742310813
Mars	−0.1324784979851	−1.3269846289449	−0.6055546775302	0.0144822029927	0.0000753473440	−0.0003541199691
Jupiter	−5.3941968663323	−0.7709883235977	−0.1989109026640	0.0010056289515	−0.0065350237136	−0.0028281094121
Saturn	7.9482624411543	4.5071593899891	1.5199489343134	−0.0031591825559	0.0043662822334	0.0019419088581
Uranus	−18.2827509982246	−0.9583685572263	−0.1615684949776	0.0001713668001	−0.0037698516627	−0.0016541985199
Neptune	−16.3717057706423	−23.7616195537846	−9.3216332089547	0.0026228079709	−0.0015329285068	−0.0006940626345
Pluto	−30.4521946670408	−0.5325021780141	9.0594118976226	0.0002820612754	−0.0031521402749	−0.0010816544638
Moon	−0.0008357031642	−0.0019854391577	−0.0010832687705	0.0005987521184	−0.0001741537135	−0.0000884771962
Sun	0.0045144118714	0.0007228284115	0.0002465910049	−0.0000002836945	0.0000051811944	0.0000022306588

rounded values were then forced into the final solution for DE 118 which then re-adjusted (slightly) the remaining parameters.

There are a number of reasons for forcing certain parameters to have specific values: the adoption of someone else's determination of the constant, the rounding of values for cosmetic reasons, the need for consistency with outside surces. The adoption of the mass of Pluto is an example of the first reason; the rounding of the planetary masses of Jupiter, Saturn and Uranus in DE 118 are examples of the second; the adoption of Fricke's values of *k* and *n* are examples of the third.

The creation of DE 118 was done partially with the intent that this ephemeris (when rotated onto the J 2000 equator and equinox) would be used for the foundation of future national almanacs. Discussions with P.K. Seidelmann of the USNO led to the decision that not all of the constants of DE 118 would conform to the adopted set of 1976 IAU astronomical constants. Such a departure is necessary in order to fit all of the observational data to their fullest accuracy. In particular, it was not possible to use the IAU set of constants and still produce acceptable solutions for the accurate Viking Lander and lunar laser-ranging data.

6.3. *The standard deviations*

The standard deviations listed in Table 14 are the formal values, straight from the least squares solutions. They are in no way intended to represent realistic uncertainties. It is well known that such a situation is often the case: formal uncertainties tend to be overly optimistic, sometimes by an order of magnitude. This is a direct result of incorrect or incomplete modeling, either through the equations of motion or through the data reductions. In the present case, some of the sources of this optimism are known. For discussions of what are believed to be realistic uncertainties associated with the planetary ephemerides, see Newhall et al. (1983) and Williams and Standish (1989).

6.4. *More recent data and ephemerides*

Since the creation of DE 118, a number of new data sets have been incorporated into the more recent ephemeris solutions at JPL. These involve both additions to the previous data as well as completely new types of data. In preparation for the Voyager encounters with the planet Uranus, a concentrated study of that planet's ephemeris has revealed significant corrections to all of the jovian planet ephemerides in DE 118. In paticular, throughout the 1980's, the latest JPL ephemeris, DE 130 (Standish, 1987), shows right ascension offsets from DE 118 amounting to $-0\farcs1$ for Jupiter, $-0\farcs2$ for Saturn and $-0\farcs4$ or so for Uranus and Neptune. In declination, the differences are less than $0\farcs1$ in general. These offsets have been substantiated by still further data and, therefore, are believed to be significant. The later progress in these areas will be reported in a future paper.

Acknowledgements and data sources. This paper has concentrated upon the observational data, one segment of the whole process of creating ephemerides. Such data, of course, would be worthless without the many other segments in which the author has been joined by colleagues to whom he is most grateful for their support and assistance: J.G. Williams, R.W. Hellings, XX Newhall, M.S.W. Keesey, J.O. Dickey, J.H. Lieske, E.L. Lau and J.D. Anderson.

The observational data determine the quality of the ephemerides. To the contributors of the data, special gratitude is given. The following is a list of the sources of the observational data used in the JPL ephemerides, up to and including DE 118/LE 62:

— Transit Circle Data: Publications of the USNO (1911–1971) punched and transmitted by C. Oesterwinter (Dahlgren), P.K. Seidelmann (USNO), D.A. O'Handley (JPL); limb corrections added by E.M. Standish (JPL); USNO Circulars transmitted by P.K. Seidelmann (USNO).

— Astrometric plate measurements of Saturn's satellites: observed, measured and transmitted by P.A. Ianna (U of Va).

— Normal Points of Neptune and Pluto: transmitted by P.K. Seidelmann (USNO).

— Radar Ranging Data: Arecibo, Haystack, Millstone, transmitted by I.I. Shapiro and G.H. Pettingill (MIT); Goldstone, transmitted by R.M. Goldstein, G.S. Downs, P.E. Reichley, R.F. Jurgens and S. Zohar (JPL).

— Mariner 9 Occultation Points: A.J. Kliore and D.L. Cain (JPL).

— Pioneer Normal Range Points: G.W. Null (JPL).

— Mariner 9 Normal Range Points: transmitted by J.F. Jordan (JPL); analysed by J.D. Anderson and E.L. Lau (JPL).

— Viking Orbiter Normal Range Points: processed and transmitted by J.F. Jordan, E.S. Christensen and C.E. Hildebrand (JPL).

— Viking Lander Range Points: transmitted by J.P. Brenkle, D.L. Cain, P.M. Eshe (JPL); calibrated by J.D. Anderson and E.L. Lau (JPL).

— Lunar Laser Ranging: observed and transmitted by J.D. Mulholland, P.J. Shelus (U. Texas); processed by J.G. Williams, J.O. Dickey and W.S. Sinclair (JPL).

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References

Anderson, J.D.: 1978, private communication

The Astronomical Almanac for the Year 1984: 1984, US Government Printing Office, Washington or H.M. Stationary Office, London

Bretagnon, P.: 1982, *Astron. Astrophys.* **114**, 278

Chapront-Touzé, M., Chapront, J.: 1983, *Astron. Astrophys.* **124**, 50

Chao, C.C.: 1970, *Jet Prop. Lab. Tech. Memo*, **391–129**

Chollet, F.: 1984, *Astron. Astrophys.* **139**, 215

Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac: 1961, H.M. Stationary Office, London

Fricke, W.: 1971, *Astron. Astrophys.* **13**, 298

Fricke, W.: 1982, *Astron. Astrophys.* **107**, L13

Ianna, P.A.: 1974, . . . , 1980, private communication

JHD: 1984, *Basis of the New Japanese Ephemeris*, Suppl. Jpn. Ephemeris for the year 1985

Kliore, A.J., Cain, D.L., Fjeldbo, G., Seidel, B.L., Sykes, M.J.: 1972, *Icarus*, **17**, 484

Lieske, J.H., Lederle, T., Fricke, W., Morando, B.: 1977, *Astron. Astrophys.* **58**, 1

Lindegren, L.: 1977, *Astron. Astrophys.* **57**, 55

Lyttleton, R.A., Cain, D.L., Liu, A.S.: 1979, *Jet Prop. Lab. Publ.* **79–85**

Moyer, T.D.: 1971, *Jet Prop. Lab. Tech. Rep.* **32–1527**

Muhleman, D.O., Anderson, J.D.: 1981, *Astrophys. J.* **247**, 1093

Newhall, X.X., Standish, E.M., Williams, J.G.: 1983, *Astron. Astrophys.* **125**, 150

Null, G.W.: 1976, private communication

Null, G.W.: 1978, *Jet Prop. Lab. Interoffice Memo.* **314.7–252** (using data element No. 360)

Schwan, H.: 1977, *Veröff. Astron. Rechen-Inst. Heidelb.* **27**

Shapiro, I.I.: 1964, *Phys. Rev. Let.* **13**, 789

Standish, E.M.: 1973, *Astron. Astrophys.* **26**, 463

Standish, E.M., Keesey, M.S.W., Newhall, X.X.: 1976, *Jet Prop. Lab. Tech. Rep.* **32–1603**

Standish, E.M.: 1982, *Astron. Astrophys.* **114**, 297

Standish, E.M.: 1985, *Numerical Planetary and Lunar Ephemerides: Present Status, Precision and Accuracies* in *Relativity in Celestial Mechanics and Astrometry*, eds. J. Kovalevsky, V.A. Brumberg, Reidel, Dordrecht, pp. 71–83

Standish, E.M.: 1987, *Jet Prop. Lab. Interoffice Memo.* **314.6–891**

Standish, E.M.: 1987, *Jet Prop. Lab. Interoffice Memo.* **314.6–891**

Standish, E.M.: 1988, *Jet Prop. Lab. Interoffice Memo* **314.6–962**

US Geological Survey: 1976–1980, *Topographic Maps of Mars*, Flagstaff, AZ

Williams, J.G.: 1984, *Icarus* **57**, 1

Williams, J.G., Standish, E.M.: 1989, *Dynamical Reference Frames in the Planetary and Earth-Moon Systems*, in *Reference Frames*, eds. J. Kovalevsky, I.I. Mueller, B. Kolaczek, Kluwer, Dordrecht

Zodiacal Catalogue: US Naval Observatory, private communication via magnetic tape