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Error Minimization of Polynomial Approximation of Delta *T*

Islam Sana[1,*], Sadiq Muhammad[1] & Qureshi Muhammad Shahid[2]
[1]*Department of Applied Physics, University of Karachi, Pakistan.*
[2]*Institute of Space and Planetary Astrophysics, University of Karachi, Pakistan.*
[*]*e-mail: ashahsana@gmail.com*

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Abstract. The difference between Universal time (UT) and Dynamical time (TD), known as Delta T (ΔT) is tabulated for the first day of each year in the Astronomical Almanac. During the last four centuries it is found that there are large differences between its values for two consecutive years. Polynomial approximations have been developed to obtain the values of ΔT for any time of a year for the period AD 1620 to AD 2000 (Meeu 2000) as no dynamical theories describe the variations in ΔT. In this work, a new set of polynomials for ΔT is obtained for the period AD 1620 to AD 2007 that is found to produce better results compared to previous attempts.

Key words. Delta T—polynomial approximation—Dynamical time—Universal time.

1. Introduction

Currently, three different time scales are in use for modern astronomy as well as for civil life (McCarthy 2004). The Universal time (UT), based on the rotation of earth around its own axis, Ephemeris time (ET), based on the revolution of the Earth in its orbit around the Sun and the Atomic time (AT) based on the quantum mechanics of atom. Each of these bases for measure of time has its history of evolution (Nelson *et al.* 2001).

Due to irregularities in the axial rotation of the Earth, the Universal time is not a uniform time (Husfeld & Kronberg 1996). Currently the Earth is slowing down and its rate is not constant, instead it has unpredictable irregularities (Meeus 1998a). Ephemeris time (ET) is not affected by the irregularities of the axial spin of the Earth, as it is evaluated from the revolution of the Earth in its orbit around the Sun in view of the principles of celestial mechanics. In this context it is clear that ET and UT are not perfectly in agreement (Seidelmann 1985). Moreover, from the theory of relativity we know that time is a dynamical quantity and it could vary from point to point in space (co-ordinated time). In 1984, the ET was replaced by Terrestrial Dynamical time (TDT) (Meeus 1998a), which is defined by an atomic clock located at the origin of a co-ordinate system for which this time is being defined. If the origin of this co-ordinate system is chosen at the centre of the Earth, we have Terrestrial Dynamic time (TDT) or if this refers to the centre of mass of the solar system we have Barycentric dynamical time (TDB) (Guinot & Seidelmann 1998). Later TDT was renamed as "Terrestrial

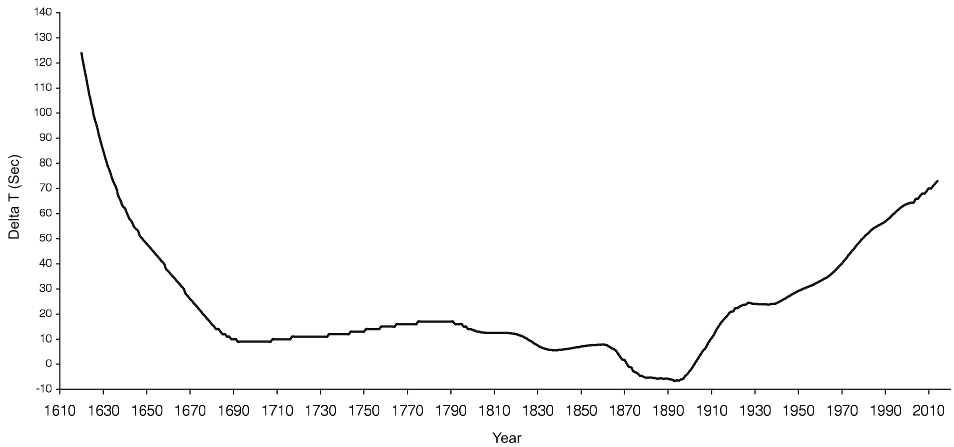

Figure 1. Shows the variation of ΔT with epoch 1620–2007.

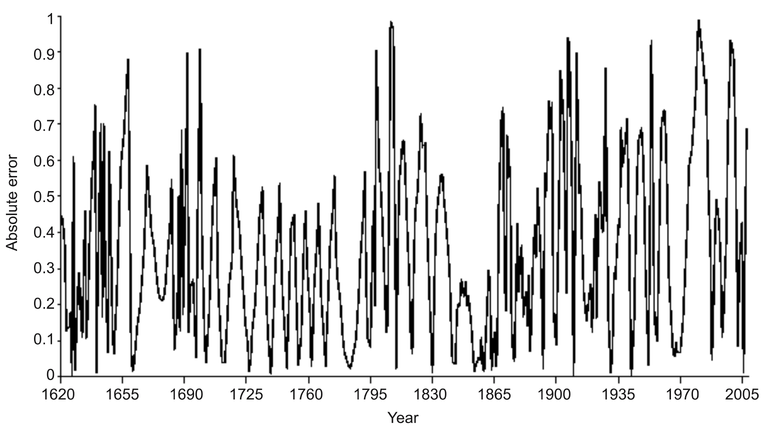

Figure 2. Shows the absolute error of new approximation.

time" (TT) (Nelson *et al.* 2001), which is now considered to be uniform time scale, and is used as the time argument for the prediction of the astronomical events. Our clocks, locked on UT are gradually slowing down with respect to the uniform TT and hence using UT, astronomical events seem to occur earlier than predicted (Seidelmann 1985). The difference between TT and UT (Meeus 1998b) is shown in Fig. 1 (Robert 2002).

2. Polynomial approximation of Delta T by Meeus

For any instant during a year one needs some approximation technique to deduce the value of ΔT. One such technique was developed by Jean Meeus and Larry Simons in 2000 that gives a set of polynomial approximation for ΔT for the range of 1620 to 2000 (Meeus & Simons 2000). It is difficult to find a single polynomial approximation for the whole span of 1620 to 2000. Meeus & Simons have divided the whole data set into eight segments/intervals (Table 1 of Meeus & Simons 2000) and determine a fourth degree polynomial for each segment that approximates ΔT for that segment.

Table 1. Depicts the co-efficient of new polynomial approximation.

Time interval (year)	k	a_0	a_1	a_2	a_3	a_4
1620–1698	3.48	38.067	−105.262	14.523	−273.116	1162.805
1699–1806	2.545	13.759	13.893	7.591	−39.048	−71.724
1807–1872	1.675	5.859	−3.654	161.524	−157.977	1612.55
1873–1906	1.175	−6.203	−2.732	139.921	1006.463	6250.501
1907–1948	0.795	24.006	12.382	−234.449	1055.209	1815.042
1949–2007	0.29	47.917	91.081	−29.979	−358.707	262.919

Using these polynomials one can calculate the value of ΔT at any instant of the year for the range 1620 to 2000 AD. Their technique is based on the formula

$$\Delta T = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4, \qquad (1)$$

where 'u' is given by

$$u = k + \frac{(\text{year} - 2000)}{100}, \qquad (2)$$

'year' could be a fractional value from 1620 to 2000 and k is a parameter that makes $u = 0$ for the middle of an interval. The set of coefficients a_i, $i = 0, 1, 2, 3, 4$ is different for different segments (Table 1 of Meeus & Simons 2000).

It is important to note that each polynomial is valid only for the period mentioned in the first column of Table 1 and if used for a different period the error could be very large. It should be noted that contrary to the claim of Meeus the largest absolute error is greater than 3 s (for 1627 the error is 3.207471). The error is very large in the beginning and then decreases as one proceeds to higher values of the year. The standard deviation of the error is 0.92546. This large error motivated us to search for a new set of polynomials for which the error does not exceed the 1 s limit. The next section shows that we have been successful in finding such an approximation.

3. New polynomial for Delta *T*

We have used the same technique as used by Meeus & Simons (2000) introducing a parameter k, which minimizes 'u' in a specified interval. The only difference is that instead of (2) we have used:

$$u = k + \frac{(\text{year} - 2007)}{100}. \qquad (3)$$

Considering polynomials of degree four we have tried to minimize the error to within ± 1 s, selecting a number of segments, i.e., time intervals with an associated polynomial to cover the whole range from AD 1620 to AD 2007. By varying the number of segments and the duration of the segments the best results were obtained for the six segments shown in Table 1 of this work as compared to the eight segments used by Meeus & Simons (2000).

The error analysis has shown that the accuracy is well within ± 1 s for the whole range of the period under consideration. The standard deviation of the error is only 0.3981 which is much better than that in the case of polynomials due to Meeus & Simons (2000). Using this set of new polynomial, the maximum error was found to be 0.990917 s for the year 1806. The major advantage of the polynomials devised by Meeus & Simons is that they give better results for recent years whereas the polynomials devised in this work provide more accurate results in general (absolute error never exceeds 1 s). In the case of Meeus & Simons (2000), the absolute error is more than 1 s in 139 cases out of 381.

References

Guinot, B., Seidelmann, P. K. 1998, *Time scales: their history, definition and interpretation, Astron. Astrophys.*, **194**, 304–308.

Husfeld, D., Kronberg, C. 1996, *Astronomical Time Keeping*, http://www.maa.mhn.de/Scholar/times.html.

McCarthy, D. D. 2004, *Precision time and the rotation of the Earth*, International Astronomical Union, Washington, DC 20392, USA, e-mail: dmc@maia.usno.navy.mil.

Meeus J. 1998a, *Astronomical algorithms*, Willmann-Bell, 2nd edn., Chapter 10 (Dynamical time and universal time).

Meeus, J. 1998b, *The effect of Delta T on astronomical calculation, Br. Asstron. Assoc.*, **108(3)**, 154–156.

Meeus, J., Simons, L. 2000, *Polynomial Approximation of Delta T, Br. Asstron. Assoc.*, **110**, 323–324.

Nelson, R. A., McCarthy, D. D., Malys, S., Levine, J., Guinot, B., Fliegel, H. F., Beard, R. L., Bartholomew, T. R. 2001, *The leap second: Its history and possible future*, *Metrologia*, **38**, 509–529.

Robert, H. 2002, *Delta T: Approximate algorithms for historical periods*, http:// www.phys.uu.nl/∼vgent/astro/deltatime.htm.

Seidelmann, P. K. 1985, *Fundamental reference system; past, present, and future celestial mechanics*, **37**, 199–207.