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ASSESSING POLYNOMIAL APPROXIMATION FOR ΔT

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ABSTRACT

The time based on the rotation of the earth around its own axis is known as Universal Time (UT), it is not a uniform time scale. Another time scale based on the revolution of earth in its orbit around the sun, called Terrestrial time (TT). The difference between TT and UT is known as delta T (ΔT). ΔT varies slowly but rather irregularly. The exact value of ΔT cannot be predicted because the rate at which earth's rotation is slowing down is not known. Therefore, delta T can only be deduced form observations. The knowledge of the exact value of ΔT is essential for predicting the correct time of astronomical event or to confirm the time of Historical events. If ΔT is not considered the result may contain an error of several seconds. The value of ΔT can be found in almanacs, where it is given in seconds of times at January 1 for each year but for some calculation of the astronomical events we require its values at any date of the year. In such cases an approximate formula can make the life simple. This paper discuss a polynomial approximation to ΔT for the range of 1620 to 2000 AD published by Jean Meeus and Larry Simons and gives a modified version of this approximation which is much more accurate.

Keywords: Polynomial Approximation for ΔT, universal time (UT), astronomical event, historical events, terrestrial time (TT), almanacs.

INTRODUCTION

Historically, we have used the repetitive cycles of celestial phenomena to serve as the basis for two components that are involved in our measurement of time. First, we make use of a phenomenon perceived to be repeatable with sufficient regularity. Second, we need to devise a convenient means to label the repetitions of this phenomenon. These elements correspond to the two properties of time measurement: interval and epoch. The repetitive celestial bodies are the moon, sun, stars etc. The rising and setting of the Sun has provided the day; the phases of the Moon have provided the month and the position of the Sun with respect to the stars gives us the year.

In the beginning, the motion of objects mentioned above forms the bases of time measurement. Nowadays three different time scales have been in use for modern astronomy as well as for civil life (Nelson, 2001). The first one is the universal time (UT), based on the rotation of earth around its own axis (McCarthy, 2004). The second is Ephemeris time (ET), based on the revolution of earth in its orbit around the sun and the third one is Atomic time (AT) based on the quantum mechanics of the atom (Nelson, 2001).

Unfortunately almost all naturally discovered periodic motions are not as simple and periodic as we thought of them earlier. It is also true in case of axial motion of earth. Therefore, Universal time (UT), as it is based on the rotation of the earth, is not a uniform time scale because the rotation of the Earth is slowing down. Moreover the rate of this slowing is not constant, instead has unpredictable irregularities (Meeus, 1998). On the other hand Ephemeris time (ET) is not affected by the irregularities of the spin motion of the earth. In this context it is clear that ET and UT are not perfectly in agreement. Moreover time is a dynamical quantity and it could vary from point to point in space (coordinated time), because of this reason in 1984 the ET was replaced by Terrestrial Dynamical time (TDT), which is define by an atomic clock located at the origin of a coordinate system for which this time is being defined (Guinot, 1998).

Now TDT, renamed as "Terrestrial Time" (TT), is considered to be uniform time scale, and used as the time argument for the predictions of the astronomical events in dynamical theories (Seidelmann, 1985). Our clocks, locked on UT are gradually slowing down with respect to the uniform TT and hence using UT, astronomical events seem to occur earlier than predicted in a uniform time scale.

The difference between TT and UT is generally called Delta T (ΔT) (Meeus, 1998, 2000, Robert, 2002). ΔT varies slowly but rather irregularly with time as shown in figure 1(Meeus 1998).

The exact value of ΔT cannot be predicted and hence can only be deduced form observations. The knowledge of the exact value of ΔT is essential for predicting the correct time of an astronomical event such as eclipses,

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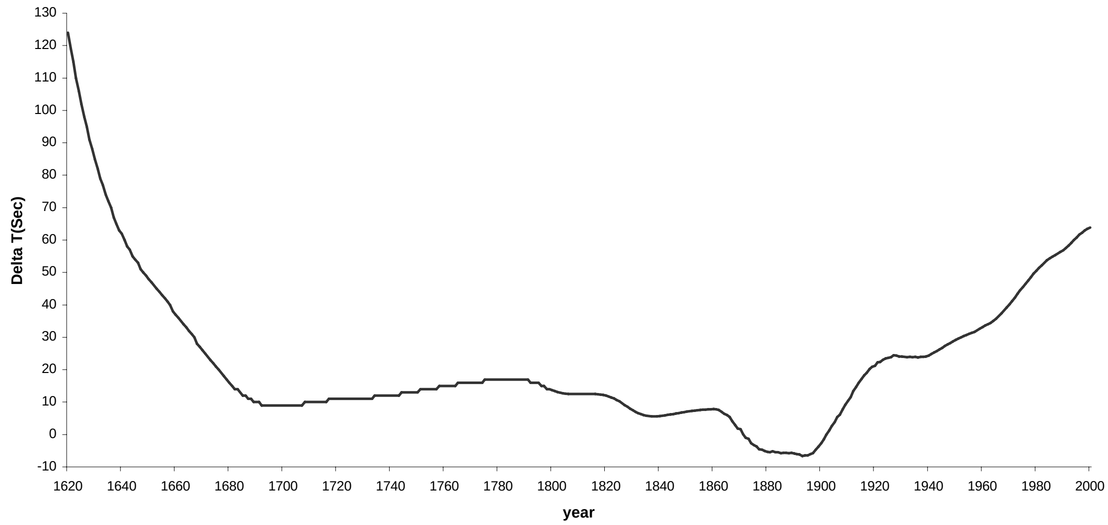

Fig. 1. Variation of ΔT between 1620 to 2000.

Table 1. Coefficient of Meeus & Simon Approximation.

Time Interval (Year)	k	a_0	a_1	a_2	a_3	a_4
1620 to 1690	3.45	40.3	-107.0	50	-454	1244
1690 to 1770	2.70	10.2	11.3	-1	-16	70
1770 to 1820	2.05	14.7	-18.8	-22	173	6
1820 to 1870	1.55	5.7	12.7	111	-534	-1654
1870 to 1900	1.15	-5.8	-14.6	27	101	8234
1900 to 1940	0.80	21.4	67.0	-443	19	4441
1940 to 1990	0.35	36.2	74.0	189	-140	-1883
1990 to 2000	0.05	60.8	82.0	-188	-5034	0

occultation, transits etc (Robert, 2002). If ΔT is not considered the predicted value may contain an error of several seconds. In some cases this error could be in minutes.

POLYNOMIAL APPROXIMATION:

1. Meeus & Simon approximation

The value of ΔT can be found in almanacs for January 1 of each year but for calculation we may require its values for any date of a year. For this purpose Jean Meeus and Larry Simons published a paper in 2000, which gives a set of polynomial approximation of ΔT for the range of years 1620 to 2000 (Meeus 2000). Meeus and Simons divided the whole curve (1620 to 2000) into eight segments and they determined a polynomial for each segment. Using these empirical formulae we can calculate the value of ΔT at any instant of the year for the range 1620 to 2000 AD. The formula to be used is

$$\Delta T = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4$$

This is a four-degree polynomial for which the coefficients (a_0 to a_4) are given in the table 1 and the 'u' is given by

$$u = k + \frac{(year - 2000)}{100}$$

The above formula shows that u is measured in centuries from the middle of the time interval mentioned in the first column of table 1.The purpose of k is simply to take the independent variable u as small as possible during that interval.

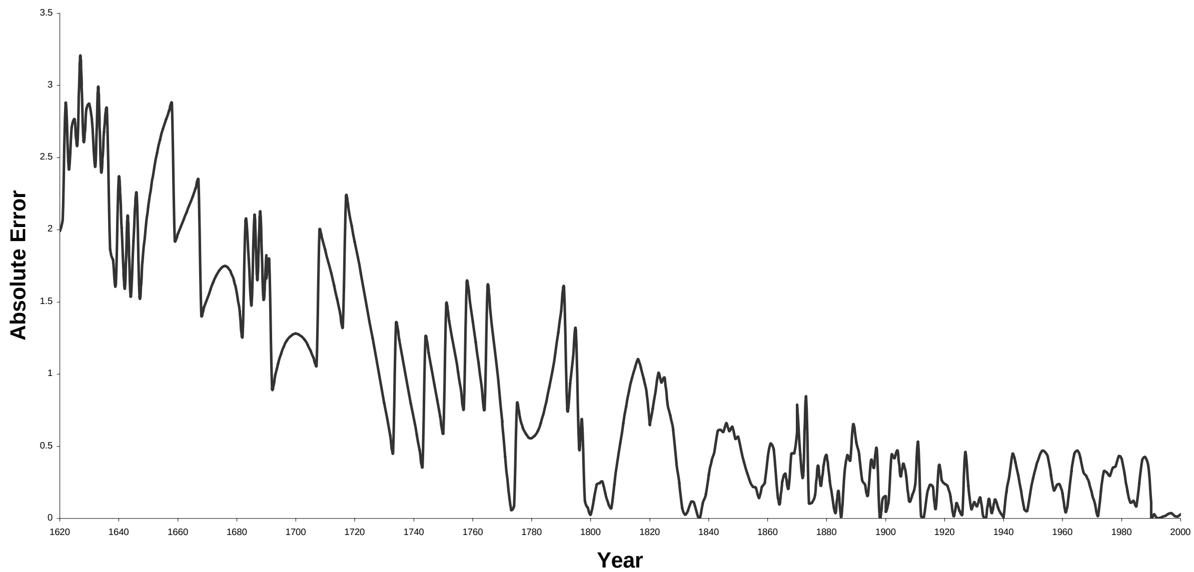

Fig. 2. Depicts the time variation of absolute data.

Table 2.Coefficient of Developed version of Approximation

Time Interval (Year)	k	a_0	a_1	a_2	a_3	a_4
1620 to 1690	3.45	42.453	-108.62	46.908	-451.441	1273.369
1690 to 1770	2.7	11.364	9.234	2.457	-1.194	45.161
1770 to 1820	2.05	15.304	-22.998	-27.101	281.575	122.178
1820 to 1870	1.55	6.085	14.218	103.619	-598.093	-1496.75
1870 to 1900	1.15	-5.571	-11.542	-40.46	-186.858	11825.13
1900 to 1940	0.8	21.462	67.422	-448.338	-11.948	4655.586
1940 to 1990	0.35	36.126	73.93	212.64	-137.364	-2383.49
1990 to 2000	0.05	60.798	81.694	-174.854	-4823.23	-2039.63

It is important to note that each expression is valid only for the period mentioned in the first column of the table 1, and if used for the different range the error could be very large. Figure 2 shows the absolute error for the whole range of 1620 to 2000. Note that the error is very large in the beginning and exceeds 3 sec. Then it is decreases as we proceed to recent years. It should be noted that initially error is very large (up to 3.2 seconds) and this large error motivated us to search for a new set of polynomials for which the error does not exceed the one-second limit. The next section shows that we have been successful in finding such an approximation.

2. Developed Version Of Approximation:

To find a new approximation which contains a maximum error of ±0.7 sec. we used least square method with a polynomial of four-degree to fit the data of figure 1. In order to compare our results with previous approximation, we keep all the parameters same as chosen by Meeus and Simons i.e. we use this algorithm for four degree and divided the whole curve of ΔT in eight segments with exactly same intervals as used by Meeus and Simons and also introduced a parameter k which minimize ‘u’ in a specified interval. The coefficients, we have calculated for the four-degree polynomial, are shown in the table 2.

Note that although these coefficients are different from Meeus & Simons approximation’s coefficients, but the procedure for finding the value of ΔT is somewhat similar in both techniques. This new approximation gives more accurate results. This accuracy is a result of the selection of more appropriate coefficients. Figure 3 shows the absolute error for the whole range of 1620 to 2000. This figure also proves that error is within ±0.7 second.

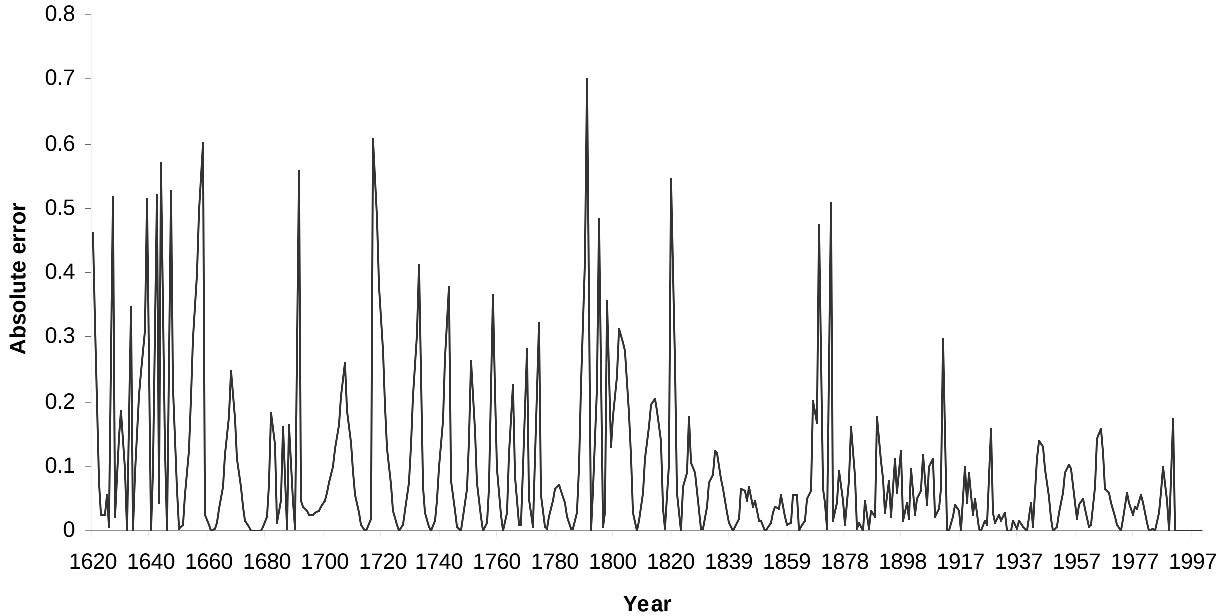

Fig. 3. Yearly variation of absolute error

To compare it with previous result lets take an example. Let's calculate the value of ΔT for the year 1627 by using both approximations, the year in the first interval so we use first polynomial. The value of 'u' is –0.28 for 1627. When we use Meeus approximation we get ΔT= 91.79sec. Whereas, we get ΔT= 94.28 sec by using this new approximation the exact value of ΔT is 95 seconds.

CONCLUSION

The above example clearly shows that new approximation gives better results. It is clearly shown by this graph that the absolute error does not exceed ±0.7 second. In fact maximum error is 0.7005 seconds, which occurs for the year 1791 AD. Now by using this approximation we can predict more accurate time for astronomical events.

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